Trade-Offs in MIMO Radar With Colocated Antennas

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SAM Workshop

Introduction

- MIMO radar is an emerging concept that has recently attracted a lot of interest
- MIMO radar: A radar system with multiple transmit waveforms that is able to jointly process signals received at multiple receive antennas
- A MIMO radar may be configured with widely separated or with co-located antennas (we focus on the latter)
- MIMO radar has many advantages such as extended array aperture, improved resolution, enhanced identifiability, etc.

Introduction (Cont'd)

- Phased-array enables coherent processing yielding an $M^2 N$ SNR gain
- As compared to phased-array, MIMO radar suffers from several drawbacks such as
  - A factor of M loss in SNR gain [Daum and Huang, 2009]
  - A factor of M loss in clear area [Abramivich and Frazer, 2008]
- Phased-MIMO radar enables applying the concept of MIMO radar without having to sacrifice the coherent processing gain

Phased-MIMO Radar

- Let $\{\phi_k(t)\}_{k=1}^{K}$ $(1 \leq K \leq M)$ be a set of baseband orthonormal waveforms
- The complex envelope of the signals transmitted by M transmit antennas

$$\underbrace{\boldsymbol{\psi(t)}}_{M\times 1} = \sqrt{\frac{M}{K}}\mathbf{W}^* \underbrace{\boldsymbol{\Phi}(t)}_{K\times 1}$$

$\boldsymbol{\Phi}(t) = [\phi_1(t), \ldots, \phi_K(t)]$: $K \times 1$ waveform vector
$\mathbf{W} = [\tilde{\mathbf{w}}_1, \ldots, \tilde{\mathbf{w}}_K]$: $M \times K$ transmit weight matrix
$\tilde{\mathbf{w}}_k$: $M \times 1$ weight vector used to form the kth Tx beam

- Tx beamforming can be optimized over $\{\tilde{\mathbf{w}}_k\}_{k=1}^{K}$ to satisfy
  - certain Tx beampattern requirements
  - total Tx power constraints
  - and/or Tx power per antenna constraints

Phased-MIMO Radar (Cont'd)

- We consider Tx array partitioning

$$\mathbf{W} \triangleq \begin{pmatrix} w_{1,1} & 0 & \cdots & 0 \\ \vdots & w_{2,1} & & \mathbf{0}_{K-2} \\ w_{1,M-K+1} & \vdots & \vdots & w_{K,1} \\ \mathbf{0}_{K-2} & w_{2,M-K+1} & & \vdots \\ 0 & \mathbf{0}_{K-2} & & w_{K,M-K+1} \end{pmatrix}$$

- Under fixed total transmit energy, the signal transmitted by the kth subarray

$$\mathbf{s}_k(qT+t) = \sqrt{\frac{M}{K}}\mathbf{w}_k^*\phi_k(t)$$

$\mathbf{w}_k$: (M-K+1)$\times$ 1 unit-norm weight vector
$E_{\mathbf{s}_k} = M/K$: total Tx energy constraint ($E_T = M$)

Phased-MIMO Radar (Cont'd)

- Signal reflected from target

$$r(qT+t) \triangleq \sqrt{\frac{M}{K}}\beta_t(q)\sum_{k=1}^{K} e^{-\jmath\tau_k(\theta_t)}\left(\mathbf{w}_k^H\mathbf{a}_k(\theta_t)\right)\phi_k(t)$$

$\beta_t(q)$: target reflection coefficient (changes from pulse to pulse)
$\tau_k(\theta_t)$: propagation time-delay between 1st element of 1st subarray and 1st element of kth subarrays

- The $N \times 1$ received data snapshot

$$\mathbf{x}(qT+t) = \sqrt{\frac{M}{K}}\beta_t(q)\sum_{k=1}^{K} e^{-\jmath\tau_k(\theta_t)}\left(\mathbf{w}_k^H\mathbf{a}_k(\theta_t)\right)\mathbf{b}(\theta_t)\phi_k(t) + \mathbf{x}_{i+n}(qT+t)$$

Phased-MIMO Radar (Cont'd)

- Using K matched filters

$$\mathbf{x}_k(q) \triangleq \int_T \mathbf{x}(qT+t)\phi_k^*(t)dt, \quad k=1,\dots,K$$

- Stacking $\{\mathbf{x}_k(q)\}_{k=1}^K$ yields the $KN \times 1$ virtual snapshot

$$\mathbf{y}(q) = \sqrt{\frac{M}{K}}\beta_t(q)\mathbf{u}(\theta_t) + \mathbf{y}_{i+n}(q) \tag{1}$$

$\mathbf{u}(\theta)$ is the $KN \times 1$ virtual steering vector

$$\mathbf{u}(\theta) \triangleq \begin{bmatrix} \mathbf{w}_1^H\mathbf{a}_1(\theta)e^{-j\tau_1(\theta)} \\ \vdots \\ \mathbf{w}_K^H\mathbf{a}_K(\theta)e^{-j\tau_K(\theta)} \end{bmatrix} \otimes \mathbf{b}(\theta). \tag{2}$$

Phased-MIMO Radar (Cont'd)

- For $K = 1$, (1) and (2) simplify to the phased-array while the case $K = M$ corresponds to MIMO radar.
- The benefits of MIMO radar can be achieved at a higher SNR gain.
- Tx beamforming can be optimized over $\{\mathbf{w}_k\}_{k=1}^{K}$ to satisfy
  - certain Tx beampattern requirements.
  - Tx power constraints, e.g. per antenna

Non-adaptive Tx/Rx Beamforming

We analyze the performance of the proposed formulations in the context of conventional Tx/Rx beamforming

- Tx/Rx beamforming weights

$$\mathbf{w}_k = \frac{1}{\sqrt{M-K+1}}\mathbf{a}_k(\theta_t)$$

$$\mathbf{w}_d = \mathbf{u}(\theta_t)$$

- Normalized Tx/Rx Beampattern

$$G_K(\theta) \triangleq \frac{\left|\mathbf{w}_d^H \mathbf{u}(\theta)\right|^2}{\left|\mathbf{w}_d^H \mathbf{u}(\theta_t)\right|^2} = \frac{\left|\mathbf{u}(\theta_t)^H \mathbf{u}(\theta)\right|^2}{\|\mathbf{u}(\theta_t)\|^2}$$

Phased-MIMO Tx/Rx Beampattern

- For simplicity, consider ULA
  - $e^{-\jmath\tau_k(\theta)} = \mathbf{a}_{[k]}(\theta)$
  - $\mathbf{w}_1^H\mathbf{a}_1(\theta) = \ldots = \mathbf{w}_K^H\mathbf{a}_K(\theta)$
- Rewriting the virtual steering vector

$$\mathbf{u}(\theta) = \begin{bmatrix} \mathbf{w}_1^H\mathbf{a}_1(\theta)e^{-\jmath\tau_1(\theta)} \\ \vdots \\ \mathbf{w}_K^H\mathbf{a}_K(\theta)e^{-\jmath\tau_K(\theta)} \end{bmatrix} \otimes \mathbf{b}(\theta) = \mathbf{w}_K^H\mathbf{a}_K(\theta)[\tilde{\mathbf{a}}(\theta) \otimes \mathbf{b}(\theta)]$$

$\tilde{\mathbf{a}}(\theta)$: $K \times 1$ vector (first K entries of $\mathbf{a}(\theta)$)
$\mathbf{a}_K(\theta)$: last $M - K + 1$ entries of $\mathbf{a}(\theta)$

Phased-MIMO Tx/Rx Beampattern (Cont'd)

Rewrite the beampattern expression as

$$G_K(\theta) = \underbrace{\frac{\left|\mathbf{a}_K^H(\theta_t)\mathbf{a}_K(\theta)\right|^2}{\|\mathbf{a}_K(\theta_t)\|^2}}_{C_K(\theta)} \cdot \underbrace{\frac{\left|\tilde{\mathbf{a}}^H(\theta_t)\tilde{\mathbf{a}}(\theta)\right|^2}{\|\tilde{\mathbf{a}}(\theta_t)\|^2}}_{D_K(\theta)} \cdot \underbrace{\frac{\left|\mathbf{b}^H(\theta_t)\mathbf{b}(\theta)\right|^2}{\|\mathbf{b}_K(\theta_t)\|^2}}_{R(\theta)} \quad (3)$$

$C_K(\theta)$: Tx coherent processing pattern
$D_K(\theta)$: waveform diversity pattern
$R(\theta)$: Rx coherent processing pattern (does not depend on K)

Phased-MIMO Tx/Rx Beampattern (Cont'd)

By inspecting (3), we have

- For $K = 1$ (Phased-array radar)

$$C_1(\theta) = \frac{\left|\mathbf{a}^H(\theta_t)\mathbf{a}(\theta)\right|^2}{\|\mathbf{a}^H(\theta_t)\|^2}, \qquad D_1(\theta) = 1$$

  Maximum Tx coherent gain, no diversity gain

- For $K = M$ (Traditional MIMO radar)

$$C_M(\theta) = 1, \qquad D_M(\theta) = \frac{\left|\mathbf{a}^H(\theta_t)\mathbf{a}(\theta)\right|^2}{\|\mathbf{a}^H(\theta_t)\|^2}$$

  No Tx coherent gain, Maximum waveform diversity gain

Phased-MIMO Output SINR

- The output SINR

$$\mathrm{SINR} \triangleq \frac{\frac{M}{K}\sigma_{\beta_t}^2 |\mathbf{w}_d \mathbf{u}(\theta_t)|^2}{\mathbf{w}_d \mathbf{R}_{i+n} \mathbf{w}_d} = \frac{\frac{M}{K}\sigma_{\beta_t}^2 |\mathbf{u}^H(\theta_t)\mathbf{u}(\theta_t)|^2}{P_i + \sigma_n^2 \|\mathbf{u}(\theta_t)\|^2}$$

- Dominant noise power ($P_i \ll \sigma_n^2$)

$$\mathrm{SINR_K} \simeq \frac{\sigma_{\beta_t}^2}{\sigma_n^2} \cdot \frac{M}{K} \|\mathbf{u}(\theta_t)\|^2 = \underbrace{M(M-K+1)N}_{\text{Gain}} \underbrace{\frac{\sigma_{\beta_t}^2}{\sigma_n^2}}_{\text{SNR}}$$

Phased-MIMO Output SINR (Cont'd)

- Dominant noise power (Cont'd)
  - Phased-array output SINR ($K = 1$)

$$\mathrm{SINR}_{\mathrm{PH}} \simeq M^2 N \frac{\sigma_{\beta_t}^2}{\sigma_n^2}$$

  - MIMO output SINR ($K = M$)

$$\mathrm{SINR}_{\mathrm{MIMO}} \simeq MN \frac{\sigma_{\beta_t}^2}{\sigma_n^2} = \frac{1}{M} \mathrm{SINR}_{\mathrm{PH}}$$

  - Phased-MIMO output SINR

$$\mathrm{SINR}_{\mathrm{PH-MIMO}} \simeq \frac{M - K + 1}{M} \times \mathrm{SINR}_{\mathrm{PH}} \tag{4}$$

Output SINR (Cont'd)

- Dominant interference power:
  - Phased-array and MIMO radars have the same same side-lobe levels, hence

$$\mathrm{SINR}_{\mathrm{MIMO}} \simeq \mathrm{SINR}_{\mathrm{PH}}$$

  - Phased-MIMO radar is shown to have

$$\mathrm{SINR}_{\mathrm{PH-MIMO}} \geq \mathrm{SINR}_{\mathrm{PH}}$$

via proving

$$\mathtt{SL}\left\{ \frac{\sin\frac{K\Omega}{2}}{K\sin\frac{\Omega}{2}} \cdot \frac{\sin\frac{(M-K+1)\Omega}{2}}{(M-K+1)\sin\frac{\Omega}{2}} \right\} \leq \mathtt{SL}\left\{ \frac{\sin\frac{M\Omega}{2}}{M\sin\frac{\Omega}{2}} \right\}$$

$\Omega \triangleq \frac{\pi d \sin(\theta)}{\lambda}$

$\mathtt{SL}\{\cdot\}$ highest sidelobe level

Phased-Array Tapering

- Consider the special case when

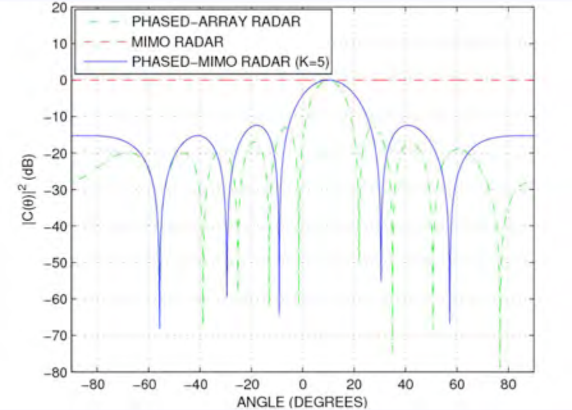
$$\phi_1(t) = \ldots = \phi_1(t) = \phi(t)$$

  can be interpreted as phased-array with tapering
- Applying the same beamforming weights per each subarray yields the same beampattern and SINR gain as that of phased-MIMO radar
- However, the waverform diversity and the corresponding benefits are lost

Example 1: Beampatterns

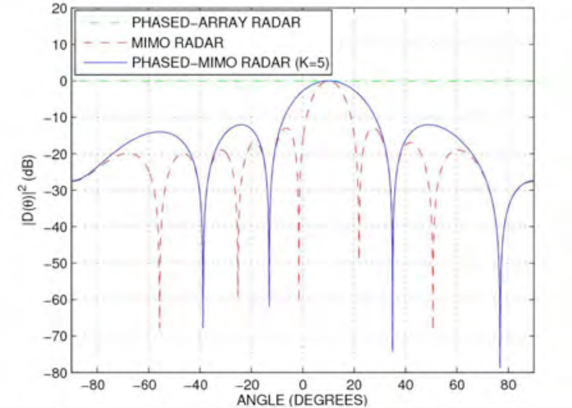
- $M = 10$ sensors spaced half a wavelength apart. Same array for Tx/Rx.
- Target located at $\theta_t = 10^\circ$
- Two interferences located at -30° and -10°.
- $K = 5$ subarrays
- $\left\{\varphi_k(t) = e^{\jmath 2\pi \frac{k}{T} t}\right\}_{k=1}^{K}$.

Example 1: Transmit Beampattern



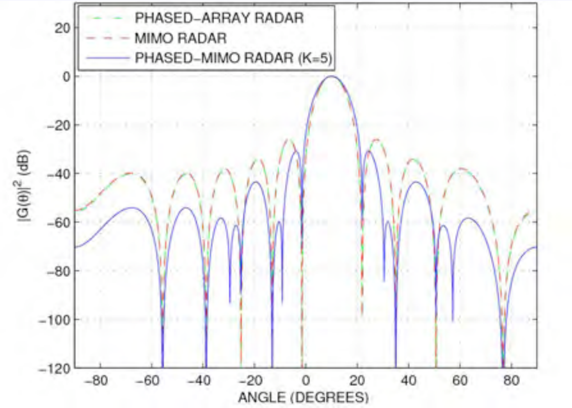
Transmit beampattern

Example 1: Diversity Beampattern



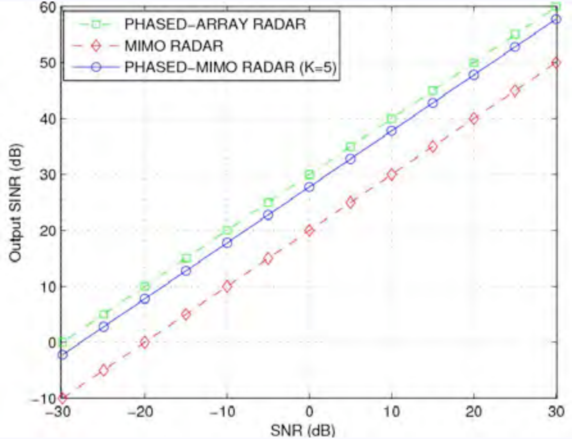
Diversity beampattern

Example 1: Overall Tx/Rx Beampattern



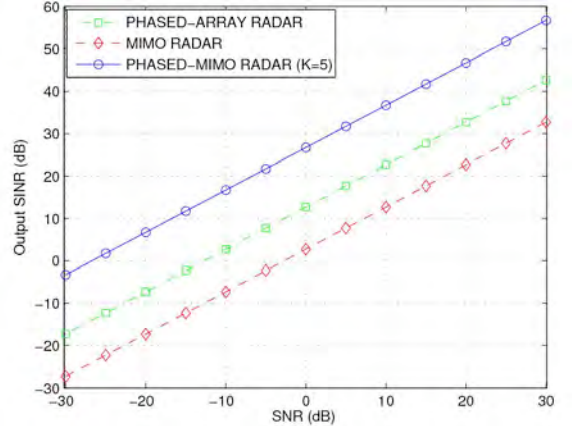
Overall Tx/Rx beampattern

Example 2: Output SINR under weak interference power



INR = -30 dB

Example 3: Output SINR under dominant interference power



INR = 30 dB

DOA Estimation in MIMO Radar

- DOA estimation for MIMO radar using ESPRIT [Duofang *et. al.*'08] and PARAFAC [Nion and Sidropoulos, ICASSP'09].
- In some applications, a priori information about the general angular source sectors is available.
- Omni-directional transmission results in waste of energy in the out-of-sector areas.
- Full-waveform diversity based MIMO radar suffers from SNR loss.

Solution

The SNR per virtual antenna can be increased by

- transmitting less waveforms of higher energy.
- focusing transmitted energy within certain spatial sectors.

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Contents

- A method for designing the transmit weight matrix, which is based on maximizing the energy transmitted within the desired spatial sector and minimizing the energy disseminated in the out-of-sector area, is developed.
- The proposed transmit energy focusing results in SNR improvement at the receive array enabling better DOA estimation performance.
- An expression for the Cramer-Rao bound that shows its dependence on the transmit weight matrix and the number of transmitted waveforms is derived.

MIMO Radar Signal Model

- Transmitting M waveforms, lth target reflection is given by

$$r_l(t,\tau) \triangleq \sqrt{\frac{E}{M}}\beta_l(\tau)\mathbf{a}^T(\theta_l)\phi(t).$$

- Assuming L targets, the $N \times 1$ received vector is modeled as

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L} r_l(t,\tau)\mathbf{b}(\theta_l) + \mathbf{z}(t,\tau).$$

- Matched-filtering the received data to the transmitted waveforms yields

$$\mathbf{y}(\tau) = \sqrt{\frac{E}{M}}\sum_{l=1}^{L}\beta_l(\tau)\underbrace{\mathbf{a}(\theta_l)\otimes\mathbf{b}(\theta_l)}_{MN\times 1}+\tilde{\mathbf{z}}(\tau). \tag{5}$$

Proposed Formulations

- The signal radiated from a beam towards the direction θ can be modeled as

$$s(t,\theta)=\sqrt{\frac{E}{K}}\mathbf{c}_k^H\mathbf{a}(\theta)\phi_k(t)$$

- Forming K transmit beams, the $N \times 1$ received vector is modeled as

$$\mathbf{x}(t,\tau)=\sqrt{\frac{E}{K}}\sum_{l=1}^{L}\alpha_l(\tau)\cdot\left(\left(\mathbf{C}^H\mathbf{a}(\theta_l)\right)^T\phi_K(t)\right)\mathbf{b}(\theta_l)+\mathbf{z}(t,\tau).$$

- Matched-filtering $\mathbf{x}_{\text{beam}}(t,\tau)$to the kth transmitted waveforms

$$\mathbf{x}_k(\tau)\triangleq\int\mathbf{x}(t,\tau)\phi_k^*(t)dt=\sqrt{\frac{E}{K}}\sum_{l=1}^{L}\alpha_l(\tau)\left(\mathbf{c}_k^H\mathbf{a}(\theta_l)\right)\mathbf{b}(\theta_l)+\mathbf{z}_k(\tau)$$

Proposed Formulations (Cont'd)

- Stacking the individual vector components into one column vector, the $KN \times 1$ virtual data vector is obtained as

$$\begin{aligned} \mathbf{y}(\tau) &\triangleq [\mathbf{x}_1^T(\tau) \cdots \mathbf{x}_K^T(\tau)]^T \\ &= \sqrt{\frac{E}{K}} \sum_{l=1}^{L} \alpha_l(\tau) \left(\mathbf{C}^H \mathbf{a}(\theta_l) \right) \otimes \mathbf{b}(\theta_l) + \tilde{\mathbf{z}}_K(\tau). \qquad (6) \end{aligned}$$

- SNR improves due to
  - $|\mathbf{c}_k^H \mathbf{a}|^2$: transmit beamforming gain.
  - E/K: increased energy of kth waveform.

Transmit Beamspace Design

- The transmit weight matrix $\mathbf{C}$ is designed based on maximizing the ratio

$$\Gamma_k \triangleq \frac{\int_T \int_\Theta \left|\mathbf{c}_k^H \mathbf{a}(\theta)\phi_k(t)\right|^2 d\theta dt}{\int_T \int_{-\pi}^{\pi} |\mathbf{c}_k^H \mathbf{a}(\theta)\phi_k(t)|^2 d\theta dt} = \frac{\mathbf{c}_k^H \left(\int_\Theta \mathbf{a}(\theta)\mathbf{a}^H(\theta) d\theta\right)\mathbf{c}_k}{\int_{-\pi}^{\pi} |\mathbf{c}_k^H \mathbf{a}(\theta)|^2 d\theta} = \frac{\mathbf{c}_k^H \mathbf{A} \mathbf{c}_k}{2\pi \mathbf{c}_k^H \mathbf{c}_k} \quad (7)$$

  $\mathbf{A} \triangleq \int_\Theta \mathbf{a}(\theta)\mathbf{a}^H(\theta) d\theta$ is a positive definite matrix.
- The maximization of the expression (7) is equivalent to maximizing its numerator while fixing its denominator.

Transmit Beamspace Design (Cont'd)

- One way to insure that $\mathbf{c}_1 \neq \mathbf{c}_2 \ldots \neq \mathbf{c}_K$ is to impose the constraint $\mathbf{C}^H\mathbf{C} = \mathbf{I}$.
- The maximization of $\{\Gamma_k\}_{k=1}^K$ subject to the constraint $\mathbf{C}^H\mathbf{C} = \mathbf{I}$ yields

$$\mathbf{C} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K]$$

  $\{\mathbf{u}_i\}_{i=1}^K$ are K principal eigenvectors of $\mathbf{A}$.
- It is worth noting that the use of principle eigenvectors for transmit beamforming has been used by [Frazer, et. al., 2007] to minimize the reactive power at the transmitter.

Transmit Beamspace Based MUSIC

- The virtual data model (6) can be rewritten as

$$\mathbf{y}(\tau) = \sqrt{\frac{E}{K}}\mathbf{V}\boldsymbol{\alpha}(\tau) + \tilde{\mathbf{z}}_K(\tau)$$

$$\boldsymbol{\alpha}(\tau) \triangleq [\alpha_1(\tau), \ldots, \alpha_L(\tau)]^T$$
$$\mathbf{V} \triangleq [\mathbf{v}(\theta_1), \ldots, \mathbf{v}(\theta_L)]$$
$$\mathbf{v}(\theta) = \left(\mathbf{C}^H\mathbf{a}(\theta)\right) \otimes \mathbf{b}(\theta).$$

- The $KN \times KN$ transmit energy focusing based covariance matrix is then given by

$$\mathbf{R} \triangleq \mathrm{E}\left\{\mathbf{y}(\tau)\mathbf{y}^H(\tau)\right\} = \frac{E}{K}\mathbf{VSV}^H + \sigma_z^2\mathbf{I}_{KN}$$

Transmit Beamspace Based MUSIC (Cont'd)

- The eigendecomposition of $\hat{\mathbf{R}}$ can be written as
$$\hat{\mathbf{R}} = \mathbf{E}_{\rm s}\mathbf{\Lambda}_{\rm s}\mathbf{E}_{\rm s}^H + \mathbf{E}_{\rm n}\mathbf{\Lambda}_{\rm n}\mathbf{E}_{\rm n}^H.$$
- The transmit energy focusing based spectral-MUSIC estimator can be expressed as
$$f(\theta) = \frac{\mathbf{v}^H(\theta)\mathbf{v}(\theta)}{\mathbf{v}^H(\theta)\mathbf{Q}\mathbf{v}(\theta)}$$
$\mathbf{Q} = \mathbf{E}_{\rm n}\mathbf{E}_{\rm n}^H = \mathbf{I} - \mathbf{E}_{\rm s}\mathbf{E}_{\rm s}^H$ is the projection matrix onto the noise subspace.
- Substituting $\mathbf{v}(\theta)$ into (31), we obtain
$$f(\theta) = \frac{N\mathbf{a}^H(\theta)\mathbf{C}\mathbf{C}^H\mathbf{a}(\theta)}{\left[\left(\mathbf{C}^H\mathbf{a}(\theta)\right) \otimes \mathbf{b}(\theta)\right]^H\mathbf{Q}\left[\left(\mathbf{C}^H\mathbf{a}(\theta)\right) \otimes \mathbf{b}(\theta)\right]}.$$

Cramer-Rao Bound

- The CRB for estimating the DOAs can be expressed as

$$\mathrm{CRB}(\theta) = \frac{\sigma_z^2 K}{2QE}\left\{\mathrm{Re}\left(\mathbf{D}^H \mathbf{P}_{\mathbf{V}}^{\perp} \mathbf{D} \odot \mathbf{G}^T\right)\right\}^{-1}$$

$$\mathbf{P}_{\mathbf{V}}^{\perp} \triangleq \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H,\ \mathbf{G} \triangleq (\mathbf{S}\mathbf{V}^H\mathbf{R}^{-1}\mathbf{V}\mathbf{S}),\ \mathbf{D} \triangleq [\mathbf{d}(\theta_1), \ldots, \mathbf{d}(\theta_L)]$$

$$\begin{aligned}\mathbf{d}(\theta) &= \frac{d\left[(\mathbf{C}^H\mathbf{a}(\theta)) \otimes \mathbf{b}(\theta)\right]}{d\theta} \\ &= (\mathbf{C}^H\mathbf{a}'(\theta)) \otimes \mathbf{b}(\theta) + (\mathbf{C}^H\mathbf{a}(\theta)) \otimes \mathbf{b}'(\theta).\end{aligned}$$

- It can also be shown that if $\mathbf{C}_{K+1} = [\mathbf{C}_K,\ \mathbf{c}_{K+1}]$ ($\mathbf{c}_{K+1}$ is a non-principal eigenvector) is used, then we have

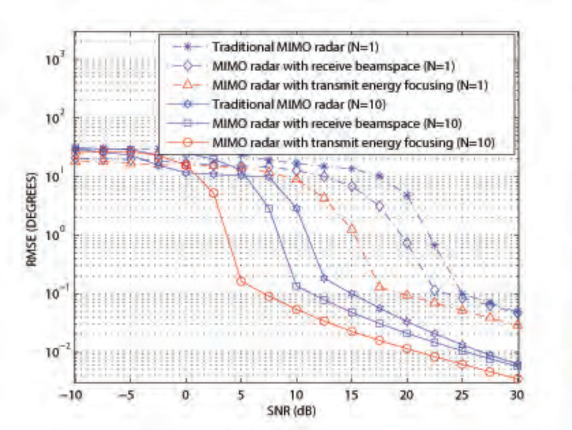
$$\mathrm{CRB}_{K+1} > \mathrm{CRB}_K.$$

Simulation Results

- ULA, $\Delta = \lambda/2$, $M = 10$, and $\boldsymbol{\Theta} = [-10^\circ \ 10^\circ]$.
- Two types of receivers (i) a single receive antenna; (ii) an arbitrary linear receive array of $N = 10$ antennas; random locations.
- Two targets are located at directions 2° and 4°.
- Fixed total transmit energy $E = M$.
- Targets are assumed to be resolved if

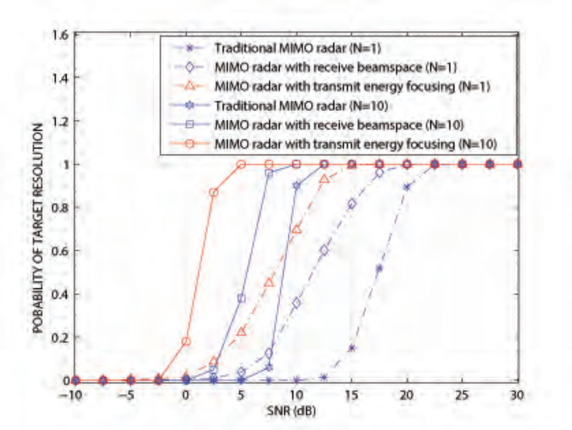
$$|\hat{\theta}_i - \theta_i| \leq \frac{|\theta_2 - \theta_1|}{2}, \quad i = 1, 2.$$

Simulation Results (Cont'd)



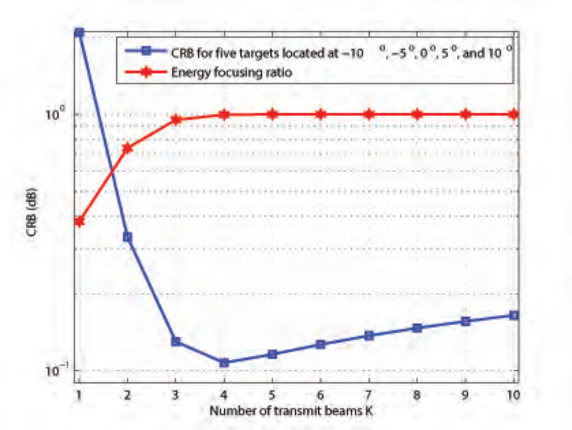
RMSE versus SNR

Simulation Results (Cont'd)



Probability of source resolution

Simulation Results (Cont'd)



CRB versus number of transmit beams K

Transmit Beamspace Signal Model

- Assume L targets are located within spatial sector $\boldsymbol{\Theta}$.
- Each transmit subarray is taken as the whole transmit array, i.e., all K subarrays are identical.
- Let $\mathbf{W}(\theta) = [\mathbf{w}_1, \dots, \mathbf{w}_K]$ be the $M \times K$ transmit beamspace matrix.
- The $M \times 1$ weight vector $\mathbf{w}_k$ can be designed to focus the energy of the kth waveform within Θ.
- The $N \times 1$ received vector

$$\mathbf{x}_{\text{beam}}(t,q) = \sqrt{\frac{E}{K}} \sum_{l=1}^{L} \beta_l(q) \mathbf{b}(\theta_l) \left(\mathbf{W}^H \mathbf{a}(\theta_l) \right)^T \boldsymbol{\Phi}(t) + \mathbf{z}(t,q).$$

$\boldsymbol{\Phi}(t) \triangleq [\phi_1(t), \dots, \phi_K(t)]^T$: $K \times 1$ waveform vector.

Transmit Beamspace Signal Model (Cont'd)

- Match-filtering to the K transmitted waveforms

$$\begin{aligned} y_k(q) &= \int_T \mathbf{x}_{\text{beam}}(t,q)\phi_k^*(t)dt, \qquad k = 1,\dots,K \\ &= \sqrt{\frac{E}{K}}\sum_{l=1}^{L}\beta_l(q)\left(\mathbf{w}_k^H\mathbf{a}(\theta_l)\right)\mathbf{b}(\theta_l) + \mathbf{z}_k(q). \end{aligned} \tag{8}$$

- Rewrite (8) as

$$\mathbf{y}_k(q) = \mathbf{B}_k\boldsymbol{\beta}(q) + \mathbf{z}_k(q). \tag{9}$$

$$\mathbf{B}_k \triangleq [\mathbf{b}(\theta_1),\dots,\mathbf{b}(\theta_L)]\boldsymbol{\Psi}_k$$
$$\boldsymbol{\Psi}_k \triangleq \mathrm{diag}\{\mathbf{w}_k^H\mathbf{a}(\theta_1),\dots,\mathbf{w}_k^H\mathbf{a}(\theta_L)\}$$
$$\boldsymbol{\beta}(q) \triangleq [\beta_1(q),\dots,\beta_L(q)]^T.$$

- The form (9) enjoys rotational invariance properties

$$\mathbf{B}_k = \mathbf{B}_j\boldsymbol{\Psi}_j^{-1}\boldsymbol{\Psi}_k, \quad k,j = 1,\dots,K.$$

Therefore, PARAFAC/ESPRIT can be applied to (8).

Transmit Beamspace Based ESPRIT

- When $K = 2$, i.e., two transmit beams

$$\mathbf{B}_2 = \mathbf{B}_1 \boldsymbol{\Psi}$$

$$\boldsymbol{\Psi} \triangleq \mathrm{diag}\left\{ \frac{\mathbf{w}_2^H \mathbf{a}(\theta_1)}{\mathbf{w}_1^H \mathbf{a}(\theta_1)}, \ldots, \frac{\mathbf{w}_2^H \mathbf{a}(\theta_L)}{\mathbf{w}_1^H \mathbf{a}(\theta_L)} \right\}. \qquad (10)$$

- Rewrite (10) as

$$\boldsymbol{\Psi} = \mathrm{diag}\left\{ A(\theta_1) e^{\jmath \Omega(\theta_1)}, \ldots, A(\theta_L) e^{\jmath \Omega(\theta_L)} \right\}$$

$$A(\theta) = |\mathbf{w}_2^H \mathbf{a}(\theta) / \mathbf{w}_1^H \mathbf{a}(\theta)|, \;\; \Omega(\theta) = \mathrm{angle}\{\mathbf{w}_2^H \mathbf{a}(\theta) / \mathbf{w}_1^H \mathbf{a}(\theta)\}.$$

- ESPRIT-based techniques can estimate $\boldsymbol{\Psi}$.

Special Case: ULA Transmit Array

- Assuming $\mathbf{w}_1 = [\tilde{\mathbf{w}}^T\, 0]^T$ and $\mathbf{w}_2 = [0\, \tilde{\mathbf{w}}^T]^T$, then

$$\tilde{\mathbf{w}}^H \mathbf{a}_2(\theta) = \tilde{\mathbf{w}}^H \mathbf{a}_1(\theta) e^{-\jmath \frac{2\pi d}{\lambda} \sin\theta}$$

  $\mathbf{a}_1(\theta)$: contains first $M-1$ of $\mathbf{a}(\theta)$,
  $\mathbf{a}_2(\theta)$: contains last $M-1$ of $\mathbf{a}(\theta)$.

- In this case,

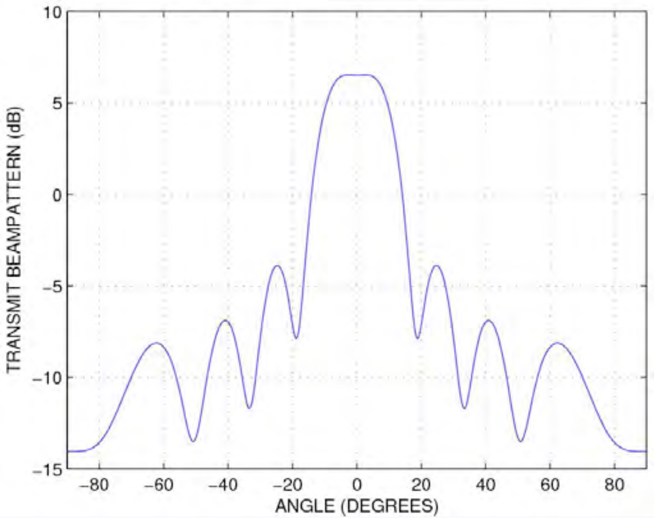
$$\mathbf{\Psi} = \mathrm{diag}\left\{ e^{-\jmath \frac{2\pi d}{\lambda} \sin\theta_1}, \ldots, e^{-\jmath \frac{2\pi d}{\lambda} \sin\theta_L} \right\}.$$

Simulation Results

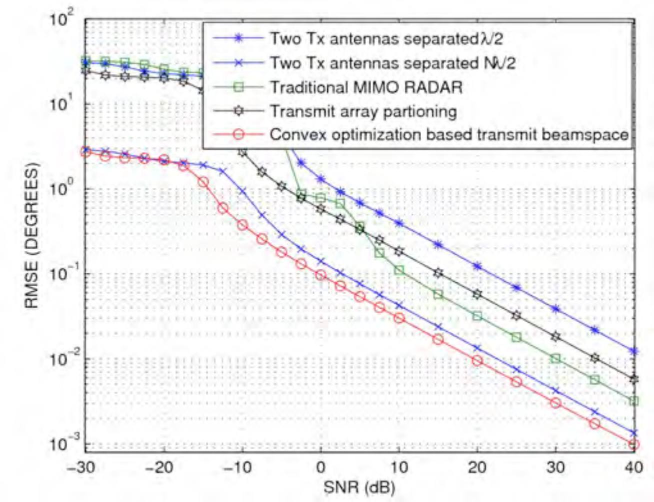
- $M = 10$ transmit antennas spaced half a wavelength apart.
- $N = 10$ receive antenna (arbitrary array)
- Two target located at $\theta_1 = 1^\circ$ and $\theta_2 = 3^\circ$
- The sector of interest is $[-5^\circ\ 5^\circ]$.
- Fixed total transmit energy $E = M$.
- Two orthogonal transmit waveforms are used.
- $\tilde{\mathbf{w}} = [-0.5623\ -0.5076\ -0.4358\ -0.3501\ -0.2542\ -0.1524\ -0.0490\ 0.0512\ 0.1441]^T$.
- Targets are assumed to be resolved if

$$|\hat{\theta}_i - \theta_i| \leq \frac{|\theta_2 - \theta_1|}{2}, \quad i = 1, 2.$$

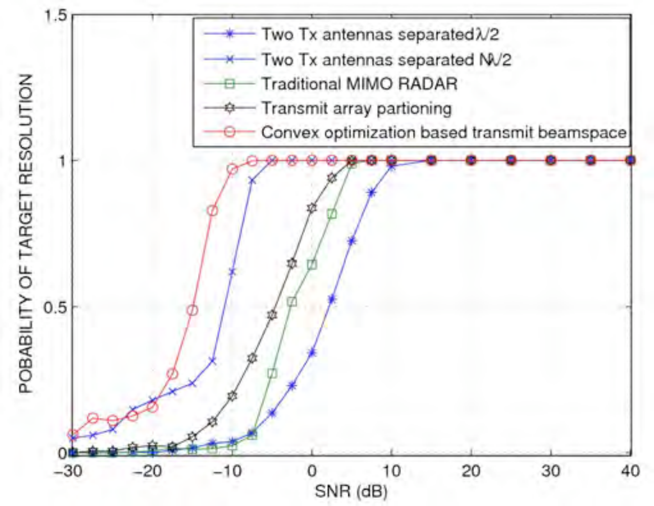
Simulation Results (Cont'd)



Simulation Results (Cont'd)



Simulation Results (Cont'd)



2D Arrays

- Sparse 2D transmit arrays are commonly used in practice to reduce the cost.
- Sparse 2D arrays are also used to realize dual-band radar systems.
- Sparse arrays do not straightforwardly enable the use of computationally efficient direction finding algorithms.

- A method for mapping an arbitrary transmit array into an array with virtually uniform structure enables
  - SNR improvement due to 2D transmit beamforming gain.
  - Root-MUSIC for computationally efficient target localization.

Signal Model and Problem Formulation

- Consider a MIMO radar with planar transmit and receive arrays of M and N antennas, respectively.
- The transmit array can be a non-uniform array or a sparse array.
- The $M \times 1$ transmit array steering vector can be represented as

$$\mathbf{a}(\theta,\phi) = \left[e^{-j2\pi\boldsymbol{\mu}^T(\theta,\phi)\mathbf{p}_1}, \ldots, e^{-j2\pi\boldsymbol{\mu}^T(\theta,\phi)\mathbf{p}_M} \right]^T$$

θ and ϕ: Elevation and Azimuth angles, respectively
$\mathbf{p}_m \triangleq [x_m \; y_m]^T, \; m = 1, \ldots, M$: Locations of the transmit antennas
$\boldsymbol{\mu}(\theta,\phi) = [\sin\theta\cos\phi \;\; \sin\theta\sin\phi]^T$: The propagation vector.

Signal Model and Problem Formulation (Cont'd)

- Let $\mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_Q]$ be the $M \times Q$ interpolation matrix $(Q \leq M)$.
- Let $\mathbf{d}(\theta, \phi)$ be the $Q \times 1$ interpolated array steering vector

$$\mathbf{C}^H \mathbf{a}(\theta, \phi) \simeq \mathbf{d}(\theta, \phi) \qquad \theta \in \boldsymbol{\Theta}, \ \phi \in \boldsymbol{\Phi}$$

- The signal radiated towards a hypothetical spatial location (θ, ϕ) is

$$\xi(t, \theta, \phi) = \mathbf{d}^T(\theta, \phi)\mathbf{s}(t) = \sum_{i=1}^{Q} \left(\mathbf{c}_i^H \mathbf{a}(\theta, \phi)\right) s_i(t)$$

$\mathbf{s}(t) = [s_1(t), \ldots, s_Q(t)]$: $Q \times 1$ vector of orthogonal wave-forms.

Signal Model and Problem Formulation (Cont'd)

- Assuming L targets, the receive observation vector is

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L} \beta_l(\tau)\Big(\mathbf{d}^T(\theta_l,\phi_l)\mathbf{s}(t)\Big)\mathbf{b}(\theta_l,\phi_l) + \mathbf{z}(t,\tau)$$

τ: Radar pulse number
$\beta_l(\tau)$: Reflection coefficient associated with l-th target
$\mathbf{b}(\theta,\phi)$: $N \times 1$ steering vector of the receive array
$\mathbf{z}(t,\tau)$: $N \times 1$ additive noise vector.

- Matched-filtering $\mathbf{x}(t,\tau)$ to $s_i(t)$ yields

$$\mathbf{y}_i(\tau) = \sum_{l=1}^{L} \beta_l(\tau) \underbrace{\Big(\mathbf{c}_i^H \mathbf{a}(\theta_l,\phi_l)\Big)}_{\text{Transmit Processing Gain}} \mathbf{b}(\theta_l,\phi_l) + \mathbf{z}_i(\tau), \quad i = 1,\ldots,Q$$

LS-based Transmit Array Interpolation

- The interpolation matrix $\mathbf{C}$ can be computed as the LS solution to

$$\mathbf{C}^H \mathbf{A} = \tilde{\mathbf{A}} \qquad (11)$$

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \ldots, \mathbf{a}(\theta_{K_\Theta}, \phi_1), \ldots, \mathbf{a}(\theta_{K_\Theta}, \phi_{K_\phi})] \quad (\mathrm{size}: M \times K_\theta K_\phi)$$

$$\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}(\theta_1, \phi_1), \ldots, \tilde{\mathbf{a}}(\theta_{K_\Theta}, \phi_1), \ldots, \tilde{\mathbf{a}}(\theta_{K_\Theta}, \phi_{K_\phi})] \quad (\mathrm{size}: \tilde{M} \times K_\theta K_\phi)$$

$\theta_k \in \mathbf{\Theta}, k = 1, \ldots, K_\theta$: Angular grid that approximates the sector $\mathbf{\Theta}$

$\phi_k \in \mathbf{\Phi}, k = 1, \ldots, K_\phi$: Angular grid that approximates the sector $\mathbf{\Phi}$.

- Given that $K_\theta K_\phi \geq M$, the LS solution to (11) can be given as

$$\mathbf{C} = \left(\mathbf{A}\mathbf{A}^H\right)^{-1} \mathbf{A}\tilde{\mathbf{A}}^H$$

This solution exhibits very high sidelobe levels resulting in wasting the transmit power within the out-of-sector regions!

Transmit Interpolation Matrix Design

- Minimize the maximum difference between the mapped array steering vector and the desired one within a certain sector $(\boldsymbol{\Theta}, \boldsymbol{\Phi})$ while keeping the sidelobe level bounded by some constant.
- The interpolation matrix design (convex) problem:

$$\begin{aligned} \min_{\mathbf{C}} \max_{\theta_k, \phi_{k'}} \quad & \left\| \mathbf{C}^H \mathbf{a}(\theta_k, \phi_{k'}) - \mathbf{d}(\theta_k, \phi_{k'}) \right\|_1 \qquad (12) \\ & \theta_k \in \boldsymbol{\Theta},\ k = 1, \ldots, K_\theta, \quad \phi_{k'} \in \boldsymbol{\Phi},\ k' = 1, \ldots, K_\phi \\ \text{subject to} \quad & \left\| \mathbf{C}^H \mathbf{a}(\theta_n, \phi_{n'}) \right\|_1 \leq \gamma, \\ & \theta_n \in \bar{\boldsymbol{\Theta}},\ n = 1, \ldots, N_\theta, \quad \phi_{n'} \in \bar{\boldsymbol{\Phi}},\ n' = 1, \ldots, N_\phi \end{aligned}$$

γ is a positive number of user choice used to upper-bound the sidelobe level.

Transmit Interpolation Matrix Design (Cont'd)

- Alternative interpolation matrix design (convex) problem:

$$\min_{\mathbf{C}} \max_{\theta_n, \phi_{n'}} \left\| \mathbf{C}^H \mathbf{a}(\theta_n, \phi_{n'}) \right\|_1 \quad (13)$$
$$\theta_n \in \bar{\boldsymbol{\Theta}},\ n = 1, \ldots, N_\theta, \quad \phi_{n'} \in \bar{\boldsymbol{\Phi}},\ n' = 1, \ldots, N_\phi$$
$$\text{subject to} \quad \left\| \mathbf{C}^H \mathbf{a}(\theta_k, \phi_{k'}) - \mathbf{d}(\theta_k, \phi_{k'}) \right\|_1 \leq \Delta$$
$$\theta_k \in \boldsymbol{\Theta},\ k = 1, \ldots, K_\theta, \quad \phi_{k'} \in \boldsymbol{\Phi},\ k' = 1, \ldots, K_\phi$$

  Δ is a positive number of user choice used to control the deviation of the interpolated array from the desired one.

- Note: Other requirements can also be enforced by imposing additional constraints in (12) and (13). Spectral Constraints [Rowe, Stoica, Li'14]

Root-MUSIC Based Localization

- Assume L-shaped desired array with half wavelength inter-element spacing $\mathbf{d}(\theta,\phi) = [\mathbf{u}^T(\theta,\phi)\ \mathbf{v}^T(\theta,\phi)]^T$

$$\mathbf{u}(\theta,\phi) = \left[1, e^{-j\pi \sin\theta\cos\phi}, \ldots, e^{-j\pi(M_d-1)\sin\theta\cos\phi}\right]^T$$

$$\mathbf{v}(\theta,\phi) = \left[1, e^{-j\pi \sin\theta\sin\phi}, \ldots, e^{-j\pi(N_d-1)\sin\theta\sin\phi}\right]^T$$

M_d: Number of desired elements on x-axis
N_d: Number of desired elements on y-axis

- Solving either (1) or (2) yields

$$\mathbf{C}^H\mathbf{a}(\theta,\phi) \approx [\mathbf{u}^T(\theta,\phi)\ \mathbf{v}^T(\theta,\phi)]^T, \quad \theta \in \mathbf{\Theta},\ \phi \in \mathbf{\Phi}$$

Root-MUSIC Based Localization (Cont'd)

- Construct the $M_d \times N$ data matrix

$$\mathbf{Y}_u(\tau) = [\mathbf{y}_1(\tau), \ldots, \mathbf{y}_{M_d}(\tau)]^T \approx \sum_{l=1}^{L} \beta_l(\tau)\mathbf{u}(\theta, \phi)\mathbf{b}^T(\theta_l, \phi_l) + \mathbf{Z}_u(\tau)$$

- Build the $M_d \times M_d$ covariance matrix

$$\hat{\mathbf{R}}_u = \sum_{\tau=1}^{T_s} \mathbf{Y}_u(\tau)\mathbf{Y}_u^H(\tau)$$

- Apply root-MUSIC to obtain the estimates

$$\zeta_l = \sin\theta_l \cos\phi_l, \ \ l = 1, \ldots, L$$

Root-MUSIC Based Localization (Cont'd)

- Similarly, construct the $N_d \times N$ data matrix

$$\mathbf{Y}_v(\tau) = [\mathbf{y}_{M_d+1}(\tau), \ldots, \mathbf{y}_{M_d+N_d}(\tau)]^T$$
$$\approx \sum_{l=1}^{L} \beta_l(\tau)\mathbf{v}(\theta, \phi)\mathbf{b}^T(\theta_l, \phi_l) + \mathbf{Z}_v(\tau)$$

- Build the $N_d \times N_d$ covariance matrix

$$\hat{\mathbf{R}}_v = \sum_{\tau=1}^{T_s} \mathbf{Y}_v(\tau)\mathbf{Y}_v^H(\tau)$$

- Apply root-MUSIC to obtain the estimates

$$\nu_l = \sin\theta_l \sin\phi_l, \ \ l = 1, \ldots, L$$

Root-MUSIC Based Localization (Cont'd)

- The ζ_l and ν_l estimates can be further arranged in the form

$$\chi_l = \zeta_l + j\nu_l, \qquad l = 1, \ldots, L$$

- The estimates of the elevation angles are obtained as

$$\hat{\theta}_l = \sin^{-1}(|\chi_l|), \; l = 1, \ldots, L$$

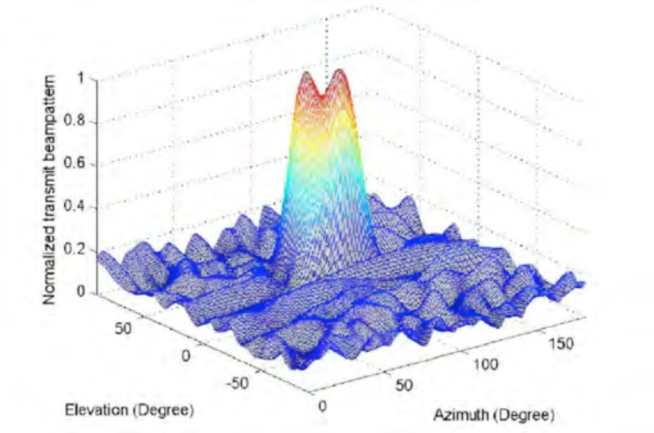
- The estimates of the azimuth angles are obtained as

$$\hat{\phi}_l = \angle(\chi_l), \; l = 1, \ldots, L$$

Simulation Results

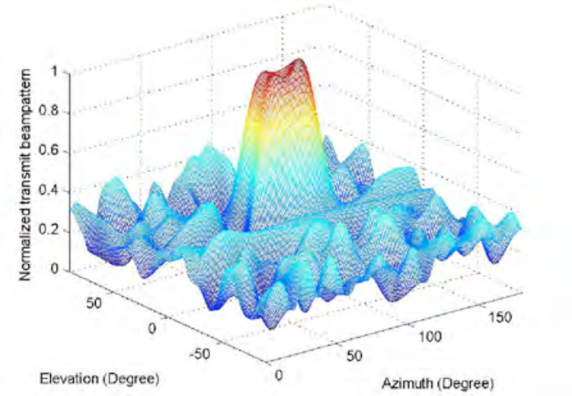
- $M = 64$ transmit antennas; x- and y-components of their positions be drawn randomly from the interval $[0, 8\lambda]$
- Transmit sector: $\mathbf{\Theta} = [30^\circ,\ 40^\circ]$ and $\mathbf{\Phi} = [100^\circ,\ 110^\circ]$
- L-shaped array: $M_d = 5$ equally spaced along the x-axis; $N_d = 5$ equally spaced along the y-axis
- Unknown target direction $\theta = 35^\circ$ and $\phi = 105^\circ$
- **C** is designed using (13) for $\Delta = 0.1,\ 0.02$, and 0.001
- $N = 16$ elements receive array (locations drawn randomly from $[0,\ 10\lambda]$)

Simulation Results (Cont'd)



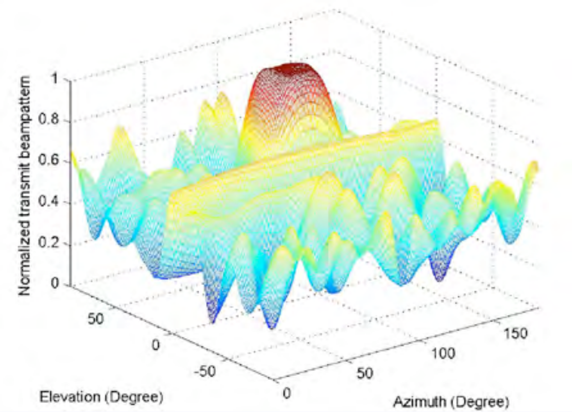
Transmit beampattern for $\Delta = 0.1$

Simulation Results (Cont'd)



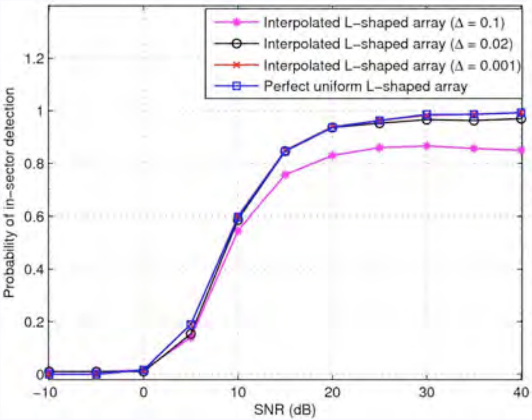
Transmit beampattern for $\Delta = 0.02$

Simulation Results (Cont'd)



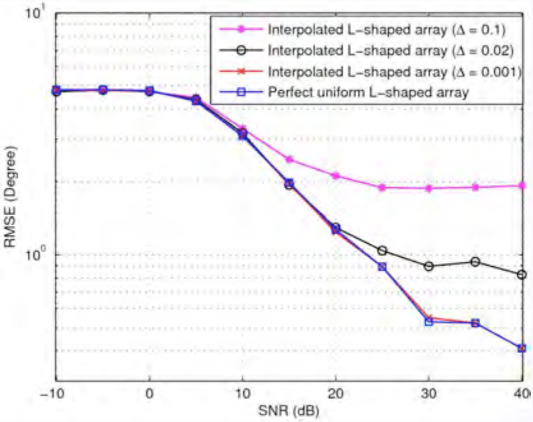
Transmit beampattern for $\Delta = 0.001$

Simulation Results (Cont'd)



Probability of in-sector detection versus SNR.

Simulation Results (Cont'd)



RMSE versus SNR.

ESPRIT-based 2D DOA Estimation

- Without loss of generality, we choose the desired array to be two perpendicular linear subarrays of two elements each.
- The virtual locations of the elements of the two subarrays are $[\tilde{x}_1,\ 0]^T$, $[\tilde{x}_2,\ 0]^T$, $[0,\ \tilde{y}_1]^T$, and $[0,\ \tilde{y}_2]^T$ ($\tilde{x}_1$, $\tilde{x}_2$, $\tilde{y}_1$, and $\tilde{y}_2$ measured in wavelength).
- Also choose the desired $\tilde{\mathbf{a}}(\theta, \phi)$ to take the following format

$$\tilde{\mathbf{a}}(\theta,\phi) = \begin{bmatrix} e^{-j2\pi\tilde{x}_1 \sin\theta} \\ e^{-j2\pi\tilde{x}_2 \sin\theta} \\ e^{-j2\pi\tilde{y}_1 \sin\phi} \\ e^{-j2\pi\tilde{y}_2 \sin\phi} \end{bmatrix}, \quad \theta \in \boldsymbol{\Theta},\ \phi \in \boldsymbol{\Phi}.$$

ESPRIT-based 2D DOA Estimation (Cont'd)

- The matched filtered virtual data at the receiver simplifies to

$$\mathbf{y}_1(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi \tilde{x}_1 \sin\theta_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_1(\tau)$$

$$\mathbf{y}_2(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi \tilde{x}_2 \sin\theta_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_2(\tau)$$

$$\mathbf{y}_3(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi \tilde{y}_1 \sin\phi_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_3(\tau)$$

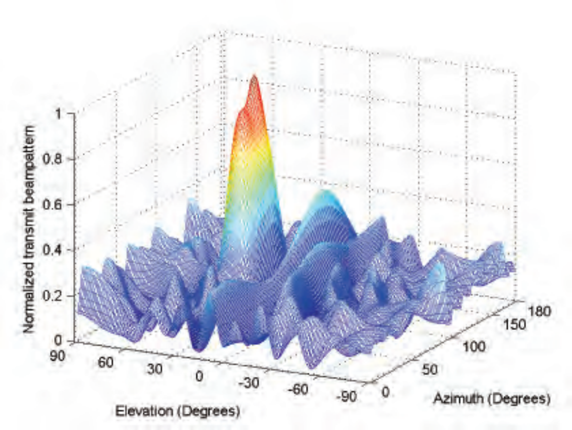
$$\mathbf{y}_4(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi \tilde{y}_2 \sin\phi_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_4(\tau)$$

- $\mathbf{y}_1$ and $\mathbf{y}_2$ enjoy rotational invariance that enables estimating $\theta_l,\ l = 1, \ldots, L$.
- $\mathbf{y}_3$ and $\mathbf{y}_4$ enjoy rotational invariance that enables estimating $\phi_l,\ l = 1, \ldots, L$.

Simulation Results

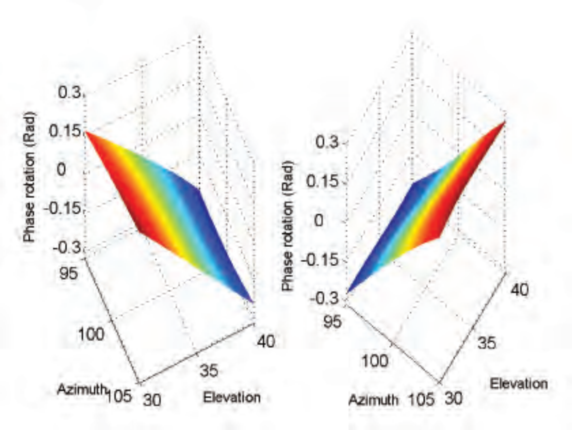
- 8×8 non-uniform rectangular transmit array
- $\mathbf{\Theta} = [30^\circ,\ 40^\circ]$ and $\mathbf{\Phi} = [95^\circ,\ 105^\circ]$
- $\tilde{x}_1 = \tilde{y}_1 = \lambda/2$ and $\tilde{x}_2 = \tilde{y}_2 = \lambda$ (λ is the wavelength)
- Two targets located $\phi_1 = 98^\circ$, $\phi_2 = 101^\circ$, $\theta_1 = 33^\circ$ and $\theta_2 = 37^\circ$
- $\mathbf{C}$ is designed using (13)–(14) using $\Delta = 0.1$
- $N = 16$ elements receive array (locations drawn randomly from $[0,\ 2\lambda]$)

Simulation Results (Cont'd)



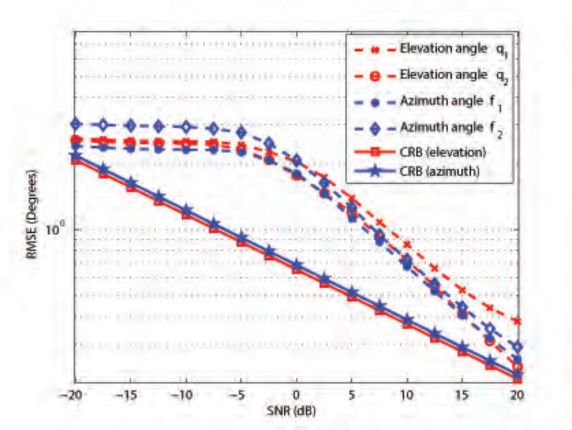
Normalized transmit beampattern

Simulation Results (Cont'd)



Left: Phase rotation between first and second elements of the interpolated arrays. Right: Phase rotation between third and forth elements of the interpolated arrays

Simulation Results (Cont'd)



RMSE versus SNR

Capability of Jammers Suppression

- **Jammers suppression** is one of the most important issues in radar signal processing.
- **MIMO radar** with colocated antennas shows advantages over phased-array (PA) radar, and provides designers with more opportunities to achieve multiple goals.
- **Capability** of efficient suppression on powerful jammers for MIMO radar has not been studied in previous works.
- **Spatial processing** techniques (with reduced dimension (RD)) such as beamspace processing or beamforming techniques significantly reduce the computational complexity.

Contents

- Three RD **beamspace designs** with robustness or adaptiveness are proposed.
- Reasonable **tradeoffs** among desired in-sector source preservation, powerful in-sector jammer suppression, and out-of-sector interference attenuation are made when designing beamspace matrices.
- Two **robust beamformers** are designed for known/unknown in-sector jammers suppression.
- Efficient **source power estimates** in the context of powerful jammers and non-ideal factors are derived.
- Capability of efficient in-sector jammers suppression using these designs is shown to be **unique** in MIMO radar.

Background

- Jamming signals take the form of high-power transmission that aim at impairing the receive system.
- Terrain-scattered jamming occurs when the high-power jammer transmits its energy to ground, and it reflects the energy in a dispersive manner.
- Jamming appears at the receive array as distributed source.
- Jammers suppression becomes more challenging when jamming impinges on the receive array from the same direction as the desired target.
- Situation worsens even more since the powerful jammers are generally unknown.

Signal Model

- L targets including the desired and interfering sources, J presumed powerful jammers
- The received signal can be modeled as

$$\mathbf{y}(\tau) = \sum_{l=1}^{L} \alpha_l(\tau)\mathbf{v}(\theta_l) + \sum_{j=1}^{J} \beta_j(\tau)\tilde{\mathbf{v}}(\theta_j) + \mathbf{z}(\tau)$$

where $\boxed{\mathbf{v}(\theta_l) \triangleq \mathbf{a}(\theta_l) \otimes \mathbf{b}(\theta_l)}$ and $\boxed{\tilde{\mathbf{v}}(\theta_j) \triangleq \mathbf{1}_M \otimes \mathbf{b}(\theta_j)}$.

- Jammer does not originate from the transmit array, therefore, does not depend on the transmit array steering vector.

RD Beamspace Processing

- RD beamspace matrix $\mathbf{B}$ of size $MN \times D$ ($D \ll MN$) with transformation

$$\tilde{\mathbf{y}}(\tau) = \mathbf{B}^H \mathbf{y}(\tau).$$

- Covariance matrix: $\mathbf{R}_{\tilde{\mathbf{y}}} \triangleq \mathbb{E}\{\tilde{\mathbf{y}}(\tau)\tilde{\mathbf{y}}^H(\tau)\} = \mathbf{B}^H \mathbf{R}_{\mathbf{y}} \mathbf{B}$
- Eigendecomposition: $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \mathbf{E}_{\mathrm{s}} \mathbf{\Lambda}_{\mathrm{s}} \mathbf{E}_{\mathrm{s}}^H + \mathbf{E}_{\mathrm{n}} \mathbf{\Lambda}_{\mathrm{n}} \mathbf{E}_{\mathrm{n}}^H$
- Beamspace spectral-MUSIC DOA estimator

$$f(\theta) = \frac{\mathbf{v}^H(\theta)\mathbf{B}\mathbf{B}^H\mathbf{v}(\theta)}{\mathbf{v}^H(\theta)\mathbf{B}\mathbf{Q}\mathbf{B}^H\mathbf{v}(\theta)}$$

$\mathbf{Q} \triangleq \mathbf{E}_{\mathrm{n}}\mathbf{E}_{\mathrm{n}}^H = \mathbf{I}_D - \mathbf{E}_{\mathrm{s}}\mathbf{E}_{\mathrm{s}}$ is the projection matrix.

Assumptions

- Desired targets are located within a known (or pre-estimated) angular sector $\boldsymbol{\Theta}$.
- Powerful jamming sources are also present within the sector-of-interest $\boldsymbol{\Theta}$, and can even have the same spatial directions as targets.
- Interfering sources are present in the out-of-sector spatial area $\bar{\boldsymbol{\Theta}}$.
- Generally, all the possible in-sector jamming and out-of-sector interfering sources are unknown.

Beamspace Design I

- Build a quiescent response beamspace matrix $\mathbf{B}_{\mathrm{q}}$ using spheroidal sequences based methods. $\boldsymbol{\Theta}$ using spheroidal sequences based methods.
- Upper-bound the acceptable difference between the desired beamspace matrix $\mathbf{B}$ and the quiescent one $\mathbf{B}_{\mathrm{q}}$ while maximizing the worst-case in-sector jammers suppression.
- Maintain out-of-sector sidelobes below a certain level.

$$\begin{aligned} \min_{\mathbf{B}} \max_{i} \quad & \left\| \mathbf{B}^H \tilde{\mathbf{v}}(\theta_i) \right\|, \ \theta_i \in \boldsymbol{\Theta}, \ i = 1, \dots, Q \\ \text{s.t.} \quad & \left\| \mathbf{B} - \mathbf{B}_{\mathrm{q}} \right\|_{\mathrm{F}} \leq \varepsilon \\ & \left\| \mathbf{B}^H \mathbf{v}(\bar{\theta}_k) \right\| \leq \gamma, \ \bar{\theta}_k \in \bar{\boldsymbol{\Theta}}, \ k = 1, \dots, K. \end{aligned} \tag{14}$$

Beamspace Design II

- Build a quiescent beamspace matrix $\mathbf{B}_{\mathrm{q}}$.
- Minimize the difference between the desired and quiescent response beamspace matrices while keeping the in-sector jammers suppression higher than a certain desired level.
- If needed, keep the out-of-sector interference attenuation at an acceptable level.

$$\begin{aligned} \min_{\mathbf{B}} \quad & \left\|\mathbf{B} - \mathbf{B}_{\mathrm{q}}\right\|_{\mathrm{F}} \\ \text{s.t.} \quad & \left\|\mathbf{B}^H \tilde{\mathbf{v}}(\theta_i)\right\| \leq \delta,\ \theta_i \in \mathbf{\Theta},\ i = 1, \dots, Q \\ & \left\|\mathbf{B}^H \mathbf{v}(\bar{\theta}_k)\right\| \leq \gamma,\ \bar{\theta}_k \in \bar{\mathbf{\Theta}},\ k = 1, \dots, K. \end{aligned} \tag{15}$$

Beamspace Design III

- Data-adaptive approach is meaningful especially when jammers and/or interfering sources are varying.
- Build a quiescent beamspace matrix $\mathbf{B}_{\mathrm{q}}$.
- Minimize output power of $\tilde{\mathbf{y}}(\tau)$ while maintaining beamspace matrix difference, in-sector jammers suppression, and out-of-sector interference attenuation below desired levels.

$$\begin{aligned}
\min_{\mathbf{B}} \quad & \mathrm{tr}\left\{\mathbf{B}^H \mathbf{R}_{\mathbf{y}} \mathbf{B}\right\} \\
\text{s.t.} \quad & \left\|\mathbf{B} - \mathbf{B}_{\mathrm{q}}\right\|_{\mathrm{F}} \leq \varepsilon \\
& \left\|\mathbf{B}^H \tilde{\mathbf{v}}(\theta_i)\right\| \leq \delta, \ \theta_i \in \mathbf{\Theta}, \ i = 1, \ldots, Q \\
& \left\|\mathbf{B}^H \mathbf{v}(\bar{\theta}_k)\right\| \leq \gamma, \ \bar{\theta}_k \in \bar{\mathbf{\Theta}}, \ k = 1, \ldots, K.
\end{aligned} \tag{16}$$

Technical Details

- Feasibility
  - (14) and (15) are always feasible for reasonable δ and γ.
  - Feasibility of (16) is guaranteed if $\varepsilon \geq \varepsilon_{\mathrm{min}}$ (minimum value of $\|\mathbf{B} - \mathbf{B}_{\mathrm{q}}\|_{\mathrm{F}}$ by solving (15)) is used for fixed δ and γ.
- Beamspace dimension
  - Depends on the width of $\mathbf{\Theta}$, no smaller than the number of principal eigenvalues of $\mathbf{A} \triangleq \int_{\mathbf{\Theta}} \mathbf{v}^H(\theta)\mathbf{v}(\theta)d\theta$.
  - No smaller than the number of desired targets for MUSIC-based DOA estimation.

Robust Beamforming I

Case I: Known Jammer Directions

- Maintain a distortionless response towards the desired target direction θ_t.
- Enforce deep nulls towards the spatial directions of the jammers.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{v}(\theta_t) = 1 \\ & \mathbf{w}^H \tilde{\mathbf{v}}(\theta_j) = 0, \ j = 1, \ldots, J \end{aligned}$$

R: $MN \times MN$ covariance matrix of interference plus jammer and noise.

Robust Beamforming II

Case II: Unknown Jammer Directions

- Enforce deep nulls towards all the possible directions of the jammers and, if needed, the out-of-sector interfering source attenuation should be kept to an acceptable level.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{v}(\theta_t) = 1 \\ & \left| \mathbf{w}^H \tilde{\mathbf{v}}(\theta_i) \right| \leq \delta, \ \theta_i \in \boldsymbol{\Theta}, \ i = 1, \ldots, Q \\ & \left| \mathbf{w}^H \mathbf{v}\left(\bar{\theta}_k\right) \right| \leq \gamma, \ \bar{\theta}_k \in \bar{\boldsymbol{\Theta}}, \ k = 1, \ldots, K. \end{aligned}$$

Source Power Estimation

- Performance degrades when array calibration errors and mismatches between the presumed and actual target steering vectors are present.
- Robust beamforming design II can be approximated by the following strengthened optimization problem

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \bar{\mathbf{v}}(\theta_t) = 1 \\ & \|\mathbf{w}^H \tilde{\mathbf{V}}\|^2 \le \tilde{\delta} \end{aligned}$$

where $\tilde{\mathbf{V}} \triangleq [\tilde{\mathbf{v}}(\theta_1), \ldots, \tilde{\mathbf{v}}(\theta_Q)]$, $\tilde{\delta} \triangleq Q\delta^2$, and $\|\bar{\mathbf{v}}(\theta) - \mathbf{v}(\theta)\|^2 \le \epsilon$.

Source Power Estimation (Cont'd)

- Closed-form solutions

  I. If $\frac{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\mathbf{R}_{\tilde{\mathbf{V}}}\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{[\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)]^2} \leq \tilde{\delta}$, then $\bar{\mathbf{w}} = \frac{\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}^H(\theta_t)}$.

  II. If $\frac{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\mathbf{R}_{\tilde{\mathbf{V}}}\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{[\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)]^2} > \tilde{\delta}$, then $\bar{\mathbf{w}} = \frac{\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}$.

- Source power estimate can be derived as

$$\bar{\sigma}_0^2 = \frac{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\mathbf{R}\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}{\left[\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)\right]^2} \qquad (17)$$

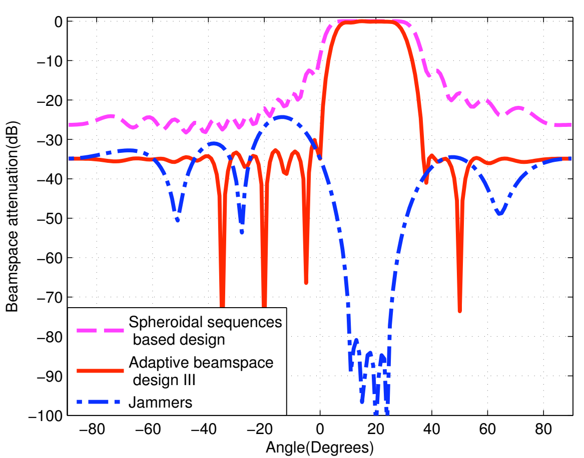
  which is valid for both conditions ($\bar{\lambda} = 0$ for the first one), and $\mathbf{R}_{\tilde{\mathbf{V}}} \triangleq \tilde{\mathbf{V}}\tilde{\mathbf{V}}^H$.

Simulation Results of Beamspace Designs

- ULA, $d_{\mathrm{T}} = d_{\mathrm{R}} = \lambda/2$, $M = 16$, $N = 8$, and $\boldsymbol{\Theta} = [10^\circ, 25^\circ]$.
- Target DOAs $\theta_t = 16.5^\circ$ and 18.5°.
- Interfering source DOAs $\theta = -35^\circ$, $-20^\circ, -5^\circ$, and 50°, respectively.
- SNR $= 0$ dB, INR $= 40$ dB, and JNR $= 50$ dB.
- $D = 7$, $\gamma = 0.2$, $\delta = 0.1$, and $\varepsilon = 1.467$.
- Targets are assumed to be resolved if

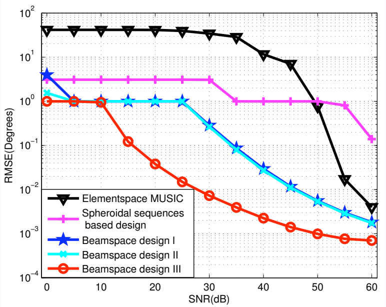
$$|\hat{\theta}_i - \theta_i| \leq \frac{|\theta_2 - \theta_1|}{2}, \quad i = 1, 2.$$

Simulation Results of Beamspace Designs (Cont'd)

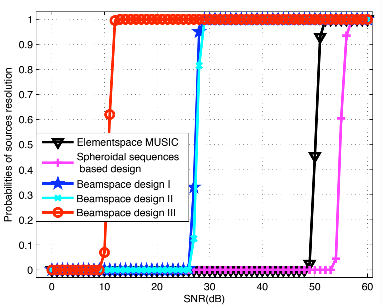


Beamspace attenuation versus angle. Uniformly distributed jammers spaced 1° apart from each other are present.

Simulation Results of Beamspace Designs (Cont'd)



(a) RMSEs of DOA estimation versus SNR.



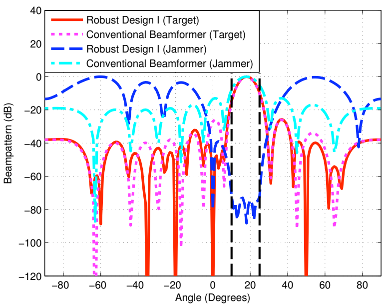
(b) Probabilities of source resolution versus SNR.

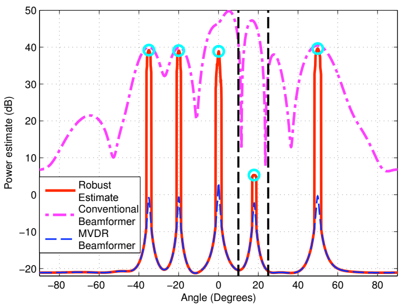
DOA estimation performance versus SNR. 5 jammers located between 15.5° and 19.5° are present.

Simulation Results of Robust Beamforming

- ULA, $d_{\mathrm{T}} = d_{\mathrm{R}} = \lambda/2$, $M = 10$, $N = 10$, and $\boldsymbol{\Theta} = [10^\circ, 25^\circ]$.
- Target DOA $\theta_t = 18^\circ$.
- Interfering sources DOAs $\theta = -35^\circ$, $-20^\circ, 0^\circ$, and 50°, respectively.
- SNR = 5 dB, INR = 40 dB, and JNR = 50 dB.
- $\tilde{\delta} = 0.0001$.

Simulation Results of Robust Beamforming (Cont'd)

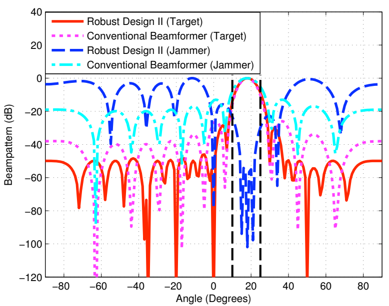




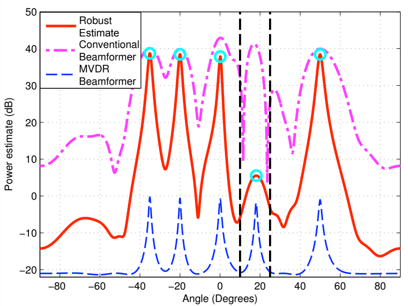
(c) Beampatterns versus angles. (d) Power estimates versus angles.

The first example. Uniformly distributed jammers spaced 1° apart from each other are present.

Simulation Results of Robust Beamforming (Cont'd)



(e) Beampatterns versus angles.



(f) Power estimates versus angles.

The second example. Three jamming signals with the DOAs $\theta_j = 15^\circ$, 18°, and 21°, respectively, are present.

Ambiguity Function of the TB-Based MIMO Radar

- **Ambiguity function** (AF) serves as a fundamental tool to evaluate radar waveform or resolution performance.
- Transmit beamspace (**TB**)**-based MIMO radar** focuses energy of multiple transmitted waveforms within a certain spatial sector where a target is likely present via TB design techniques.
- **Essence** of the TB-based MIMO radar is to achieve improved SNR gain and increased aperture simultaneously.
- Superior **DOA estimation performance** in a wide range of SNRs can be achieved in the TB-based MIMO radar.

Contents

- AF for the TB-based MIMO radar for the case of far-field targets and narrow-band waveforms is **derived**.
- Equivalent transmit phase centers are **introduced** in the AF definition.
- It serves as a **generalized** AF form for the existing radar configurations including phased-array (PA) radar and traditional MIMO radar (with subarrays).
- Relationships between the defined TB-based MIMO radar AF and the previous AF works are **established**.

Background

- Woodward's AF serves as the groundbreaking work on AF.
- Traditional MIMO radar AF becomes invalid for TB-based MIMO setup since TB processing allows non-orthogonal or correlated waveforms to be emitted at each antenna.
- Waveform correlation matrix design for desired transmit beampattern can be simplified to TB matrix design in traditional MIMO radar.
- In-depth study on TB-based MIMO radar AF may also provide insight into facilitating the clutter/interference mitigation in airborne MIMO radar system using TB design.

Signal Model

- K (in general, $K \leq M$) initially orthogonal waveforms are emitted.
- K transmit beams that fully cover the pre-estimated spatial sector $\boldsymbol{\Omega}$ are formed using TB matrix $\mathbf{C} \triangleq [\mathbf{c}_1, \ldots, \mathbf{c}_K]$.
- Signal model
  Signal radiated via the kth beam:

$$s_k(t) = \sqrt{\frac{E}{K}} \mathbf{c}_k^T \mathbf{a}(\theta) \phi_k(t), \; k = 1, \ldots, K.$$

  Signal radiated from the mth antenna:

$$\tilde{s}_m(t) = \sqrt{\frac{E}{K}} \sum_{k=1}^{K} c_{mk} \phi_k(t), \; m = 1, \ldots, M.$$

Signal Model (Cont'd)

- Let the received signal at the jth antenna be $\hat{r}_j(t, \boldsymbol{\Theta})$.
- Matched filtered noise-free signal component w.r.t. the ith waveform is

$$\begin{aligned}
\bar{r}'_{ji}(\boldsymbol{\Theta}, \boldsymbol{\Theta}') &= \int \hat{r}_j(t, \boldsymbol{\Theta}) \phi_i^*(t, \boldsymbol{\Theta}')\, dt \\
&= \sqrt{\frac{E}{K}} \sum_{m=1}^{M} \sum_{k=1}^{K} \alpha_{mj} \int c_{mk} \phi_k\big(t - \tau_{mj}(\mathbf{p})\big) \phi_i^*\big(t - \tau_{q(i)j}(\mathbf{p}')\big) \\
&\qquad \times \exp\big\{-j2\pi \tau_{mj}(\mathbf{p})\big(f_c + f_{mj}(\boldsymbol{\Theta})\big)\big\} \\
&\qquad \times \exp\big\{j2\pi \tau_{q(i)j}(\mathbf{p}')\big(f_c + f_{q(i)j}(\boldsymbol{\Theta}')\big)\big\} \\
&\qquad \times \exp\big\{j2\pi \big(f_{mj}(\boldsymbol{\Theta}) - f_{q(i)j}(\boldsymbol{\Theta}')\big) t\big\}\, dt.
\end{aligned}$$

AF Definition

- **Define** the AF as the square of coherent summation of all noise-free matched filter output pairs, i.e.,

$$\chi(\boldsymbol{\Theta},\boldsymbol{\Theta}') \triangleq \left|\sum_{j=1}^{N}\sum_{i=1}^{K}\bar{r}'_{ji}(\boldsymbol{\Theta},\boldsymbol{\Theta}')\right|^2$$

$$= \left|\sum_{j=1}^{N}\sum_{i=1}^{K}\sum_{m=1}^{M}\alpha_{mj}[\mathbf{R}]_{mi}(\boldsymbol{\Theta},\boldsymbol{\Theta}',\mathbf{C},j)\exp\{-j2\pi\tau_{mj}(\mathbf{p})\right.$$

$$\left.\times(f_c+f_{mj}(\boldsymbol{\Theta}))\}\exp\{j2\pi\tau_{q(i)j}(\mathbf{p}')(f_c+f_{q(i)j}(\boldsymbol{\Theta}'))\}\right|^2$$

AF Definition (Cont'd)

$$[\mathbf{R}]_{mi}(\boldsymbol{\Theta}, \boldsymbol{\Theta}', \mathbf{C}, j) \triangleq \sqrt{\frac{E}{K}} \sum_{k=1}^{K} c_{mk} \int \phi_k(t - \tau_{mj}(\mathbf{p})) \times \phi_i^*(t - \tau_{q(i)j}(\mathbf{p}')) \exp\{j2\pi(f_{mj}(\boldsymbol{\Theta}) - f_{q(i)j}(\boldsymbol{\Theta}'))t\} dt$$

- ϕ_k: the kth orthogonal waveform.
- f_c: radar operating frequency.
- f_{mj}: Doppler shift due to the (m, j)th transmit-receive channel.
- τ_{mj}: two-way time delay due to the (m, j)th channel.
- $q(i)$: the ith equivalent transmit phase center.

AF Illustration

- Defined AF is composed of summation terms.
- Each term contains three components:
  - Complex reflection coefficient
  - Match-filtered component standing for waveform correlation
  - Exponential terms standing for phase shift information due to relative position and target motion w.r.t. array geometry.
- Defined AF means that the mth antenna emits a signal consisting of all K initially orthogonal waveforms windowed by elements of the mth row in $\mathbf{C}$.

AF Simplification

- Neglect complex coefficients (with constant contributions).
- **Simplify** AF definition as

$$\chi(\boldsymbol{\Theta}, \boldsymbol{\Theta}') = \left| \mathbf{a}_{\mathrm{R}}^{H}(\boldsymbol{\Theta}) \mathbf{a}_{\mathrm{R}}(\boldsymbol{\Theta}') \right|^2 \left| \mathbf{a}_{\mathrm{T}}^{H}(\boldsymbol{\Theta}) \bar{\mathbf{R}}(\Delta\tau, \Delta f_d, \mathbf{C}) \mathbf{a}_{\mathrm{TE}}(\boldsymbol{\Theta}') \right|^2$$

  - $\mathbf{a}_{\mathrm{T}}$, $\mathbf{a}_{\mathrm{R}}$, $\mathbf{a}_{\mathrm{TE}}$ are the transmit, the receive, and the equivalent transmit array steering vectors, respectively.
  - $\bar{\mathbf{R}}$ derives from $\mathbf{R}$ with $\Delta\tau$ and Δf_d being used.

$$\left[\bar{\mathbf{R}}\right]_{mi}(\Delta\tau, \Delta f_d, \mathbf{C}) = \sqrt{\frac{E}{K}} \sum_{k=1}^{K} c_{mk} \int \phi_k(t) \phi_i^*(t - \Delta\tau) \exp\{j2\pi\Delta f_d t\} dt$$

Relationships With Other AFs

- Express the simplified AF using **Woodward**'s AF:

$$\chi(\boldsymbol{\Theta}, \boldsymbol{\Theta}') = \frac{E}{K}\left|\mathbf{a}_{\mathrm{R}}^{H}(\boldsymbol{\Theta})\mathbf{a}_{\mathrm{R}}(\boldsymbol{\Theta}')\right|^{2}\left|\mathbf{a}_{\mathrm{T}}^{H}(\boldsymbol{\Theta})\mathbf{C}\bar{\chi}(\Delta\tau, \Delta f_d)\mathbf{a}_{\mathrm{TE}}(\boldsymbol{\Theta}')\right|^{2}.$$

- **Traditional MIMO Radar**: $K = M$, $\mathbf{C} = \mathbf{I}_M$, $\mathbf{a}_{\mathrm{TE}} = \mathbf{a}_{\mathrm{T}}$ (uniform subarrays: $\mathbf{C}$ is block diagonal, choose subarray centers for $\mathbf{a}_{\mathrm{TE}}$)

$$\chi_{\mathrm{MIMO}}(\boldsymbol{\Theta}, \boldsymbol{\Theta}') = \frac{E}{M}\left|\mathbf{a}_{\mathrm{R}}^{H}(\boldsymbol{\Theta})\mathbf{a}_{\mathrm{R}}(\boldsymbol{\Theta}')\right|^{2}\left|\mathbf{a}_{\mathrm{T}}^{H}(\boldsymbol{\Theta})\bar{\chi}(\Delta\tau, \Delta f_d)\mathbf{a}_{\mathrm{T}}(\boldsymbol{\Theta}')\right|^{2}.$$

- **PA Radar**: $K = 1$, $\mathbf{C} = \mathbf{w}_M$ (beamforming weights), $\mathbf{a}_{\mathrm{TE}} = 1$

$$\chi_{\mathrm{PA}}(\boldsymbol{\Theta}, \boldsymbol{\Theta}') = E\left|\mathbf{a}_{\mathrm{R}}^{H}(\boldsymbol{\Theta})\mathbf{a}_{\mathrm{R}}(\boldsymbol{\Theta}')\right|^{2}\left|\mathbf{a}_{\mathrm{T}}^{H}(\boldsymbol{\Theta})\mathbf{w}_M\bar{\chi}(\Delta\tau, \Delta f_d)\right|^{2}.$$

Weak and Strong “Clear Region” Bounds

- Worst-Case Bound

$$C^{\mathrm{I}}(A) \leq \frac{4\,V_K}{\frac{N^2 K}{\left|\mathbf{a}_{\mathrm{R}}^H(\boldsymbol{\Theta})\mathbf{a}_{\mathrm{R}}(\boldsymbol{\Theta}')\right|^2} V_K - 4\eta} \tag{18}$$

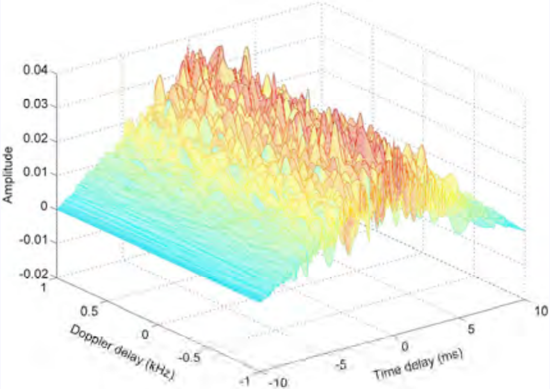
- Best-Case Bound

$$C^{\mathrm{II}}(A) \leq \frac{4\,V_K}{\frac{N^2}{\left|\mathbf{a}_{\mathrm{R}}^H(\boldsymbol{\Theta})\mathbf{a}_{\mathrm{R}}(\boldsymbol{\Theta}')\right|^2} V_K - 4\eta} \tag{19}$$

Simulation Results

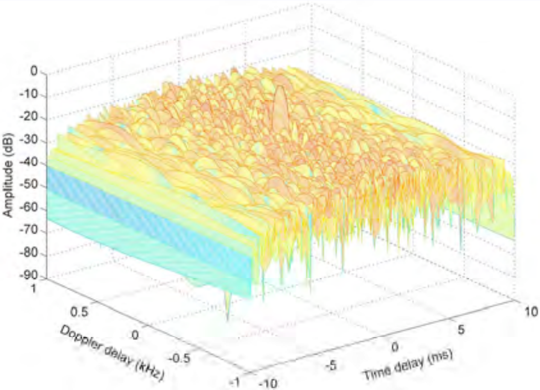
- ULA, $d_{\mathrm{T}} = d_{\mathrm{R}} = \lambda/2$, $M = 8$, $N = 8$, and $E = M$.
- Array locates on x-axis.
- Polyphase-coded waveforms with code length being 256.
- $T = 10$ ms, $BT = 128$, and $f_s = 2B$.
- Single pulse.
- Two targets locate on y-axis.

Simulation Results (Cont'd)



Difference between the defined TB-based MIMO radar AF and the squared-summation-form traditional MIMO radar AF, 8 polyphase-coded waveforms are employed.

Simulation Results (Cont'd)



TB-based MIMO radar AF, 4 polyphase-coded waveforms are employed.

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Trade-Offs in MIMO Radar with widely-spaced antennas

Marco Lops

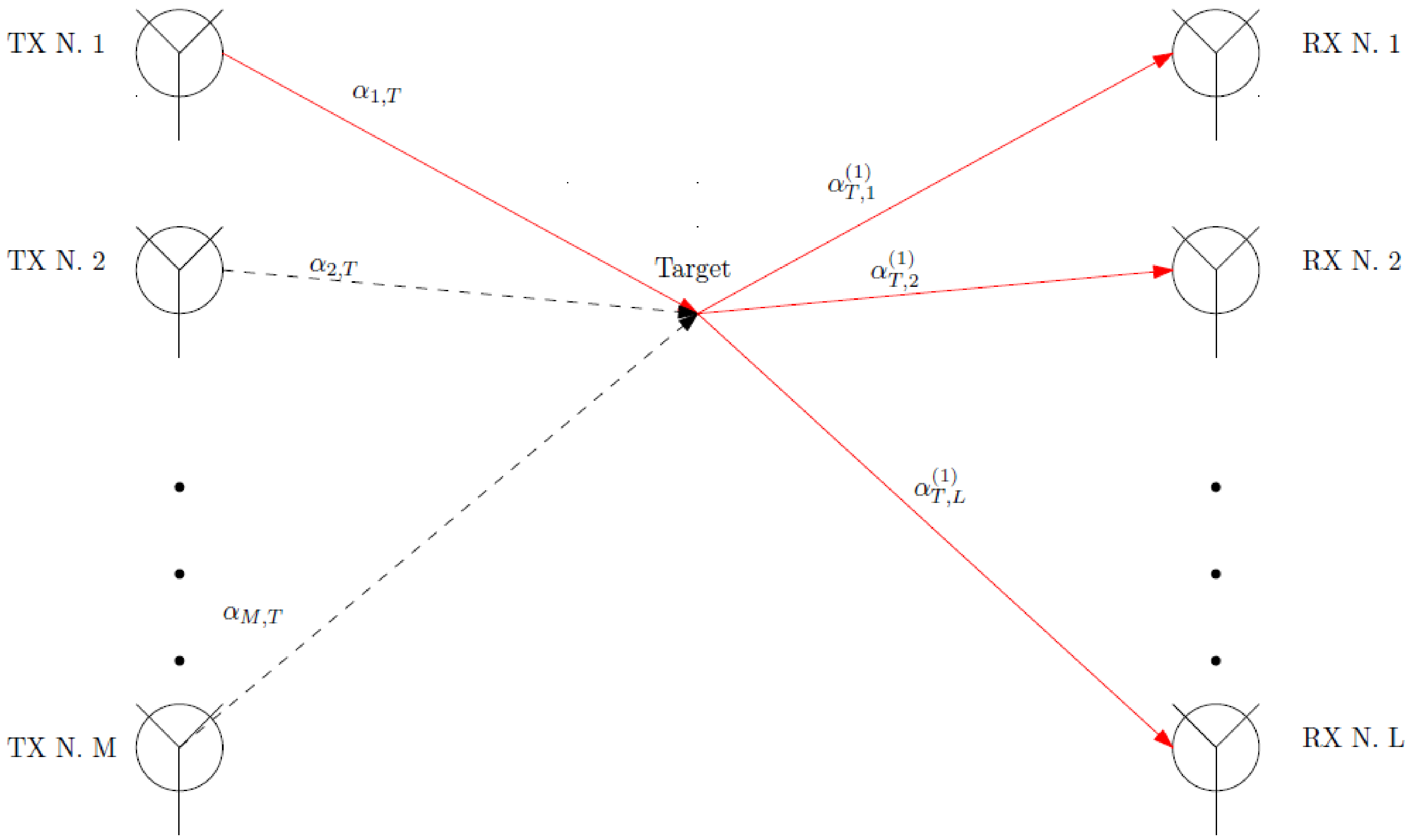
Outline

- MIMO Radar with widely spaced antennas: Angular diversity and its consequences;
- Signal models, diversity, Space-Time Coding;
- Detection through MIMOs: cost functions, figures of merit and tradeoffs;
- Robust waveform design;
- Localization through MIMO radars: should we really maximize angle diversity?
- Conclusions

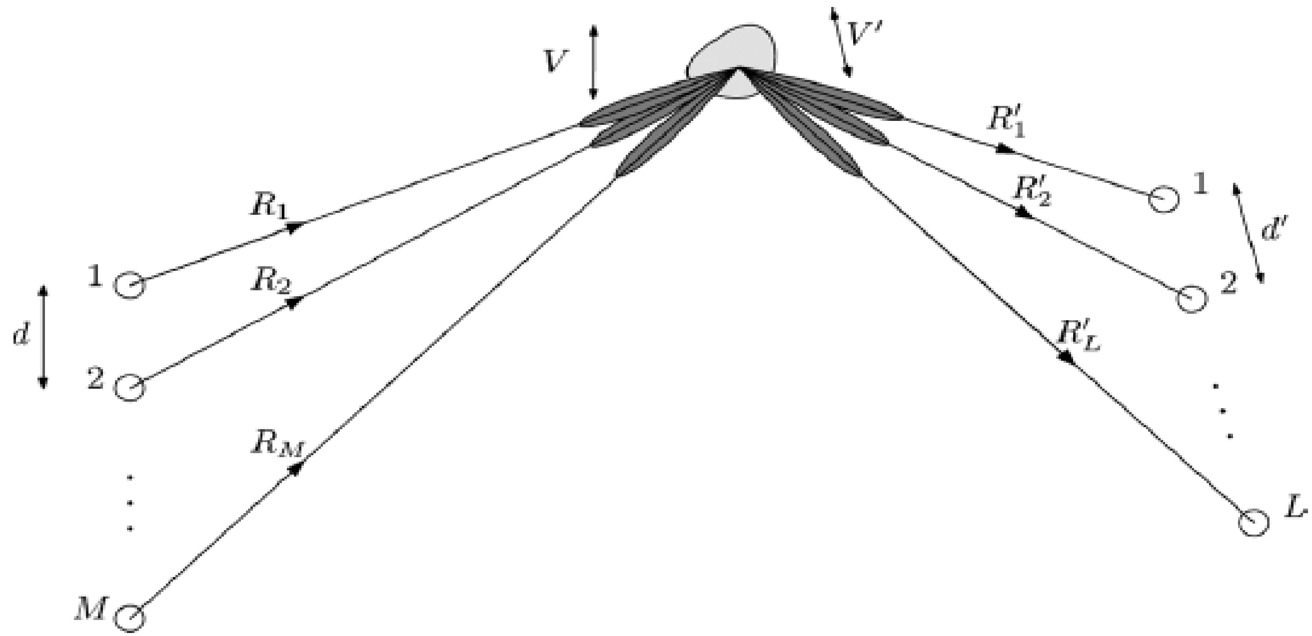
MIMO Radar

- Decentralized systems: multiple receivers and a fusion center collecting either raw data or a quantized version thereof;
- Multi-site Radars: systems joining in the network, with no real transmitter cooperation;
- Communication-theory-related MIMO: Multiple transmitters, cooperating to form the radiated waveforms and operating under power constraints, along with multiple receivers, cooperating in order to make final decisions.

Aspect angles



Transmit/Receive Diversity

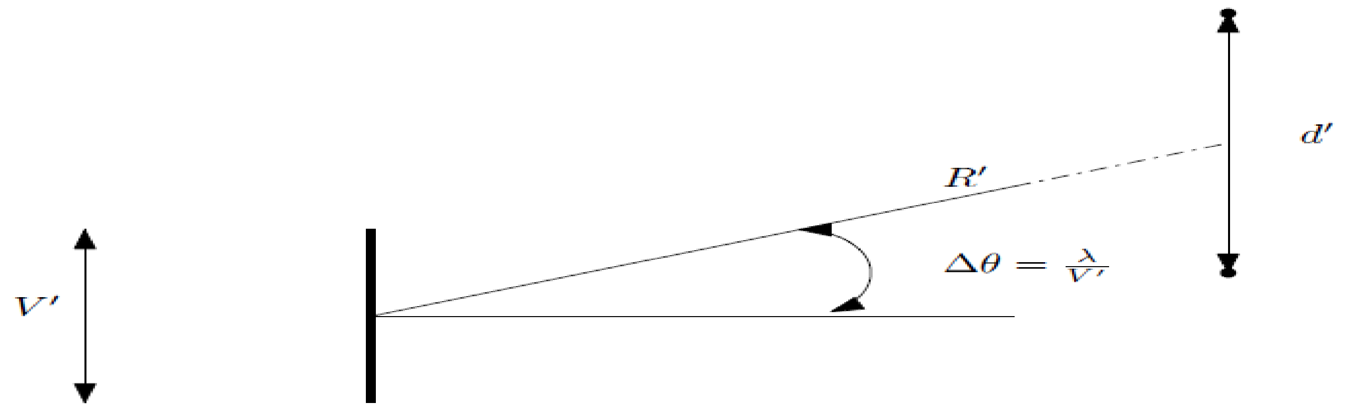


Receive diversity: $\lambda R'_\ell / V' \ < \ d'$

Transmit diversity: $\lambda R_m / V < d$

Interpretation: Receive diversity

A target whose size in the receive array alignment direction is V' becomes an aperture antenna whose beam has width λ/V'.
The scattering towards two elements spaced d' apart is independent if they are not seen as belonging to the same cell.



Arc length at distance R': $\frac{\lambda}{V'}R'$

The two receivers belong to different beams iff $R'\Delta\theta < d'$

Remark: The interpretation of transmit diversity is the same.

Consequence

The same target may not offer the same amount of angle diversity over time. Indeed, the number of independent paths turns out to depend on the target orientation (i.e., on its size along the transmit array direction) and on its range. Additionally, the same target may offer:

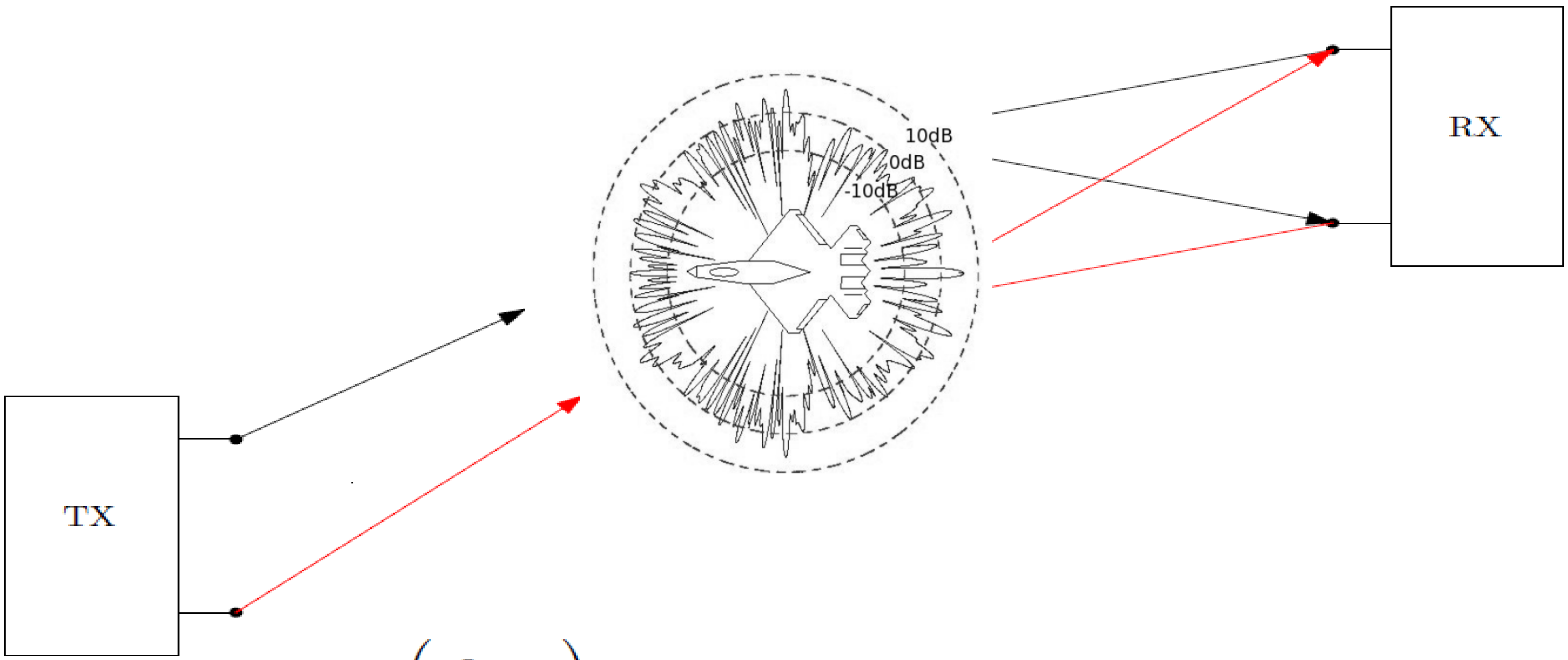
- ❑Full transmit and/or receive diversity;
- ❑Full transmit diversity and partial receive diversity (or vice-versa);
- ❑Partial (or no) diversity (transmit and/or receive).

The issue of designing waveforms which are robust with respect to the amount of real diversity is central in MIMO radar. For a physical (electromagetic) study of the relationship between target scattering and available d.o.f. see, e.g., M. D. Migliore, "Some physical limitations in the performance of statistical multiple-input multiple-output RADARs", *IET Microw. Antennas and Propagations*, vol. 2, N. 8

Angular diversity: models

- Full transmit diversity: $\alpha_{1,T}$, $\alpha_{2,T}$, ... $\alpha_{M,T}$ are statistically independent;
- Receive diversity: $\alpha^{(1)}_{T,1}$, $\alpha^{(1)}_{T,2}$, ... $\alpha^{(1)}_{T,L}$, are statistically independent, and so are $\alpha^{(i)}_{T,1}$, ... $\alpha^{(i)}_{T,L}$ for i=2,...,M
- Full diversity: both transmit and receive diversity;
- Notice that the amount of diversity varies with V, V', the target distance, and the array elements spacing.

Full diversity



$$\boldsymbol{\alpha}_1 = \begin{pmatrix} \alpha_{1,1} \\ \alpha_{2,1} \end{pmatrix} \text{ Scattering towards receive antenna 1}$$

$$\boldsymbol{\alpha}_2 = \begin{pmatrix} \alpha_{1,2} \\ \alpha_{2,2} \end{pmatrix} \text{ Scattering towards receive antenna 2}$$

The matrix $\boldsymbol{\alpha}=(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)$ has rank 2 almost surely.

Signal models

Let $\{s_i(t)\}_{i=1,\dots,M}$ the signals transmitted by the M transmit antennas

$\alpha_{i,j}$, $i = 1, \dots, M$: scattering towards receiver j from the M TX antennas

In the presence of a target, the signal received at antenna j reads:

$$\Sigma_{i=1}^{M} \alpha_{i,j} s_i(t - \tau_{i,j}) + w_j(t)$$

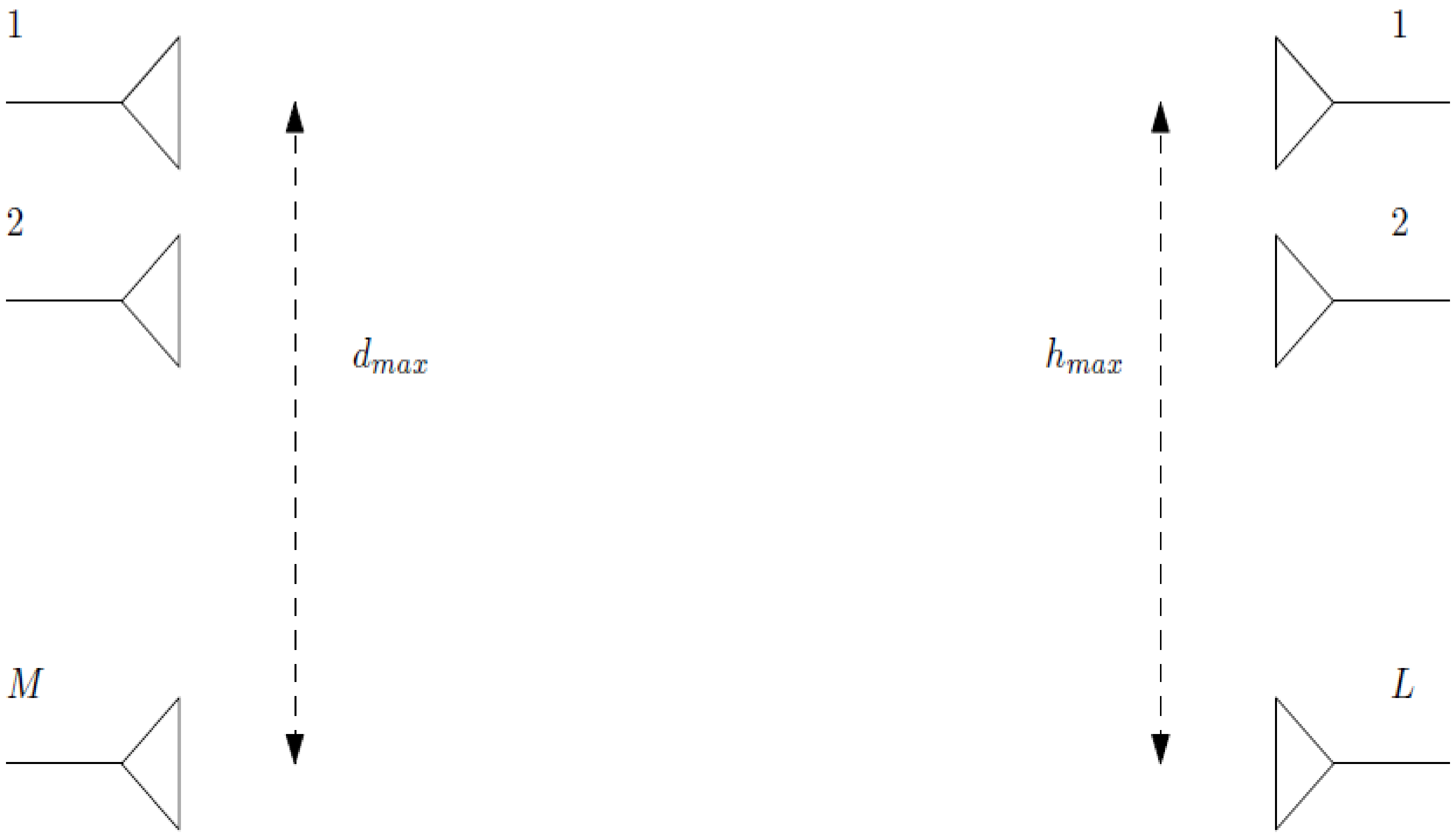
$\tau_{i,j}$: the delay between transmitter i and receiver j

$w_j(t)$: disturbance impinging on antenna j

About the delays

- Resolvable paths: the delays are all resolvable, which means that the available transmit bandwidth is large enough as to allow resolvability for most of the distances.
- Un-resolvable paths: the radar is “narrow-band”, whereby the target appears in one and the same range cell.
- The most interesting situation to study the trade-offs is the latter, as will be shown later.

Un-resolvable paths: conditions



Un-resolvable paths: $d_{max} + h_{max} << \frac{c}{B}$, c the light speed and B the bandwidth
Example: for $B = 1$ MHz the delay estimation accuracy is in the order of $1\mu s$
range resolution $\simeq 75$m $\longrightarrow d_{max} + h_{max} << 300$m.

Resolvable paths

In this case the following “orthogonality” is advocated

$$\int s_i(t-\tau_{i,j})s_k(t-\tau_{h,k})dt = (s_i(t-\tau_{i,j}), s_k(t-\tau_{h,k})) = 0 \ \forall i,j,h,k$$

This requires either extremely large bandwidths, or extremely wide spacing or radars operating on disjoint spectra. It is reminiscent more of frequency diversity than of space diversity.

Choosing the waveforms: Space-Time coding

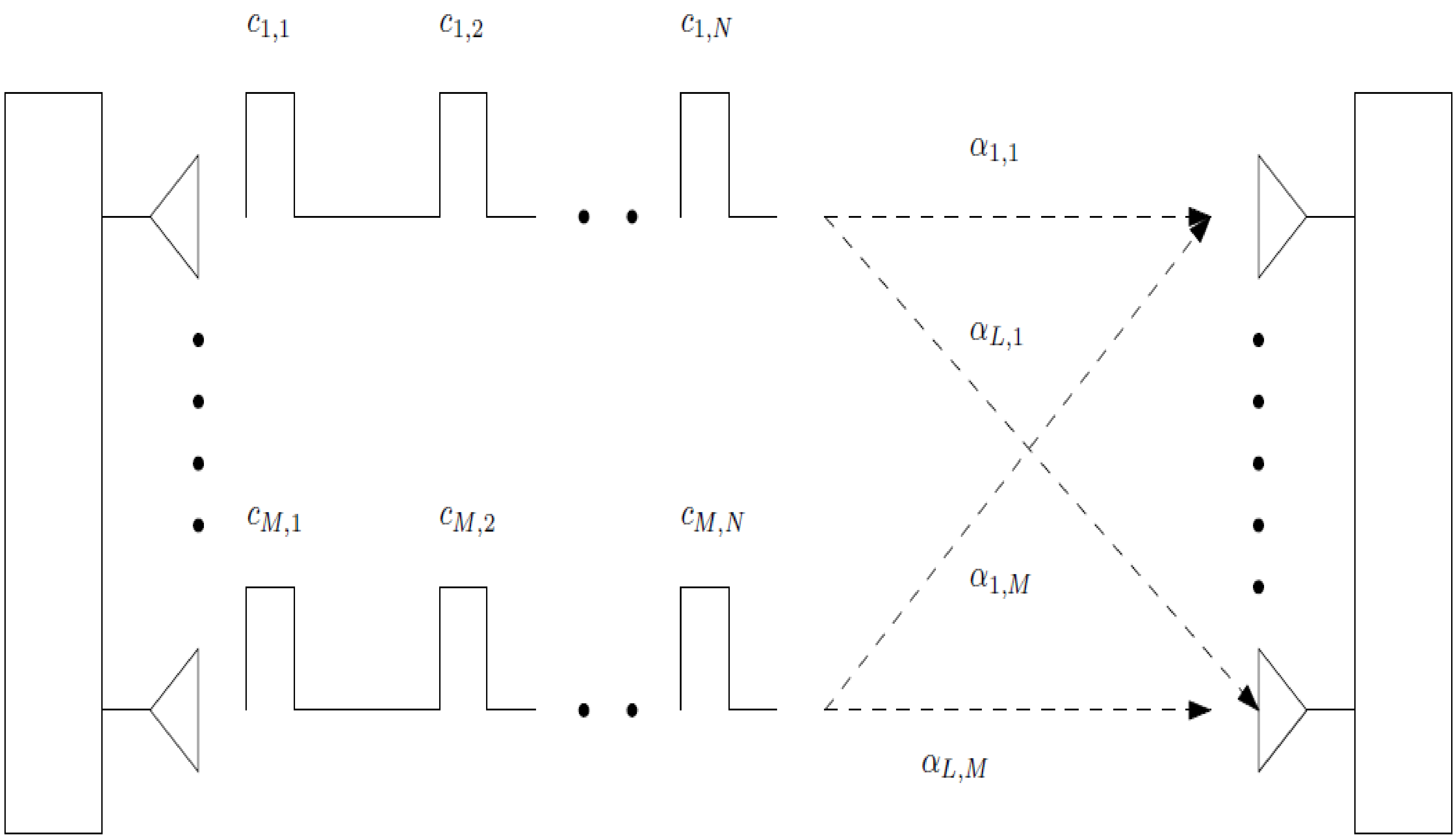
A viable way to design the waveforms is to set

$$s_i(t) = \Sigma_{n=1}^{N} c_{i,n}\psi_n(t)$$

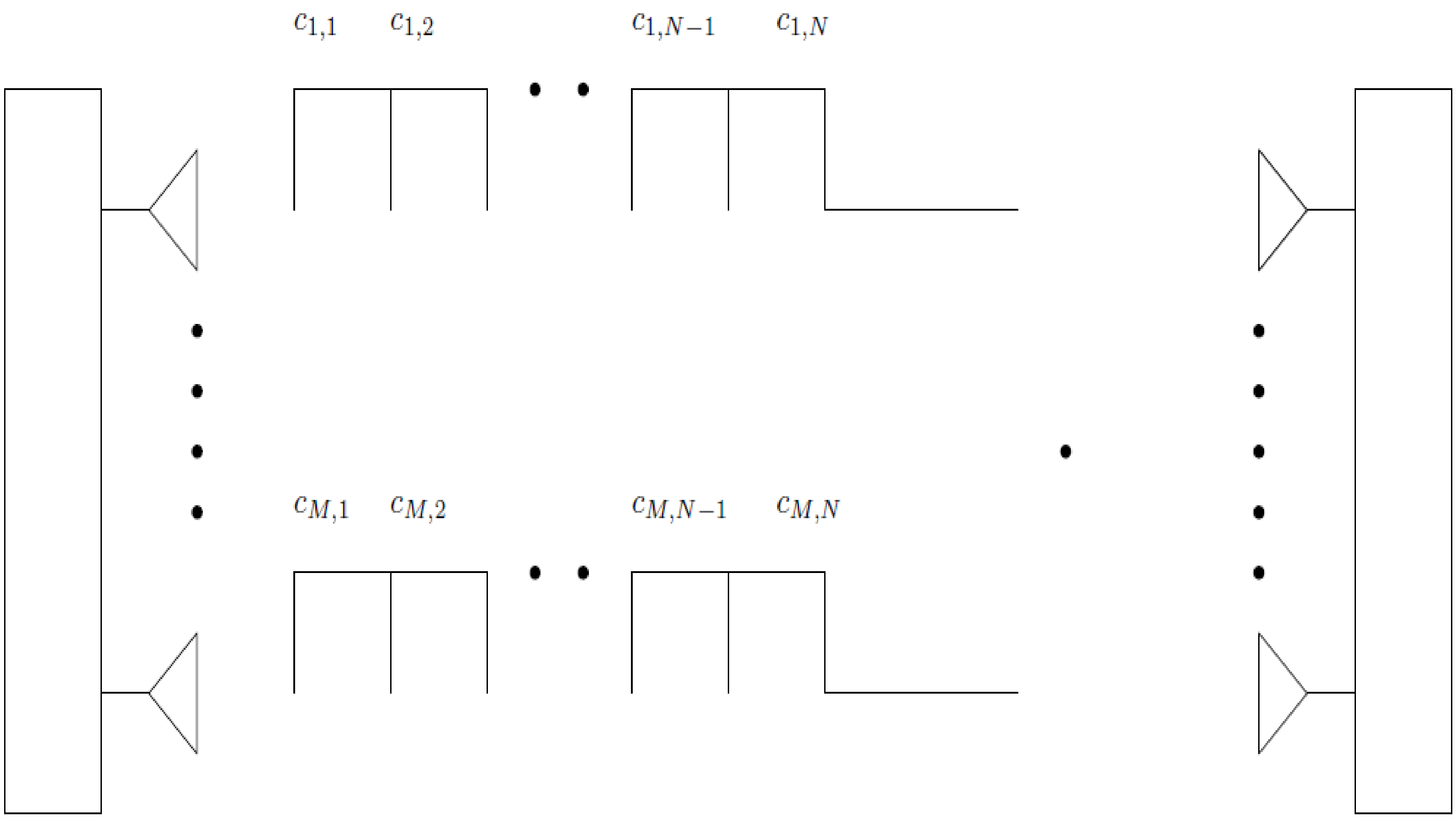
$\boldsymbol{c}_i = (c_{i,1}, \ldots, c_{i,N})$ an $N-$ dimensional codeword belonging to R^N or C^N

$\{\psi_i(t)\}$, a set of linearly independent (typically orthogonal) waveforms

Example#1: Pulsed STC radar



Example 2: Sophisticated STC Radar



The STC Matrix

The key parameter is the STC matrix

$$C = \begin{pmatrix} c_{1,1} & c_{2,1} & \cdots & c_{M,1} \\ c_{1,2} & c_{2,2} & \cdots & c_{M,2} \\ \cdots & \cdots & \cdots & \cdots \\ c_{1,N} & c_{2,N} & \cdots & c_{M,N} \end{pmatrix} = (c_1 c_2 \ldots c_M) \in \mathcal{C}^{N \times M}$$

Example: Alamouti 2×2 coding

$$C = \begin{pmatrix} c & c^* \\ -c^* & c \end{pmatrix}$$

It may be worth noticing that the simplest case is to make *c* real.

Key parameters

- Transmitted energy, encapsulated in trace($\boldsymbol{C}\boldsymbol{C}^{H}$): it represents an overall constraint that applies to the transmitted waveforms and allows performing comparisons and eliciting trade-offs;
- Rank($\boldsymbol{C}$), i.e. the rank of the STC matrix, encapsulating the transmit policy: for both resolvable - where it equals the number of active antennas- and un-resolvable paths it determines how many degrees of freedom we choose to exploit a priori.

Degrees of freedom

The matrix ***C*** defines the degrees of freedom, which are obviously

$$L\min(N,M) = \begin{cases} LM & M \leq N \quad \text{Tall} \quad \text{Matrix} \\ LN & M > N \quad \text{Fat} \quad \text{Matrix} \end{cases}$$

The reason is that, for fat matrix, the code-words are no longer linearly independent. As a consequence, for un-resolvable paths the signal space is obviously of reduced dimension.

From now on we consider the far more challenging (as far as trade-offs are concerned) situation of un-resolvable paths, since most of the conclusions, at least for target detection, carry over to the case of resolvable paths.

Target detection: signal model

The signal received at receive antenna i, $\boldsymbol{r}_i \in \mathcal{C}^N$, reads:

$$\boldsymbol{r}_i = \begin{cases} \boldsymbol{C}\boldsymbol{\alpha}_i + \boldsymbol{w}_i & \text{Target present} \quad (H_1) \\ \boldsymbol{w}_i & \text{Target absent} \quad (H_0) \end{cases}$$

Where $\boldsymbol{\alpha}_i$ is the scattering towards antenna i (and thus has dimension M) while $\boldsymbol{w}_i$ represents the impinging disturbance, which is assumed Gaussian with covariance matrix

$$\mathrm{E}\left[\boldsymbol{w}_i \boldsymbol{w}_i^H\right] = \boldsymbol{M}$$

Memo: symbol meaning

$$\boldsymbol{r}_i = \begin{pmatrix} r_{i,1} \\ \cdot \\ \cdot \\ r_{i,N} \end{pmatrix}$$ Signal received at antenna i

$$\boldsymbol{C} = \begin{pmatrix} c_{1,1} & c_{2,1} & \cdots & c_{M,1} \\ c_{1,2} & c_{2,2} & \cdots & c_{M,2} \\ \cdots & \cdots & \cdots & \cdots \\ c_{1,N} & c_{2,N} & \cdots & c_{M,N} \end{pmatrix}$$ codeword k as k-th column

$$\boldsymbol{\alpha}_i = \begin{pmatrix} \alpha_{i,1} \\ \cdot \\ \cdot \\ \alpha_{i,M} \end{pmatrix}$$ contains the scattering towards antenna i

$$\boldsymbol{w}_i = \begin{pmatrix} w_{i,1} \\ \cdot \\ \cdot \\ w_{i,N} \end{pmatrix}$$ contains the disturbance affecting antenna i

Generalized Likelihood Ratio Test (GLRT, scattering as nuisance)

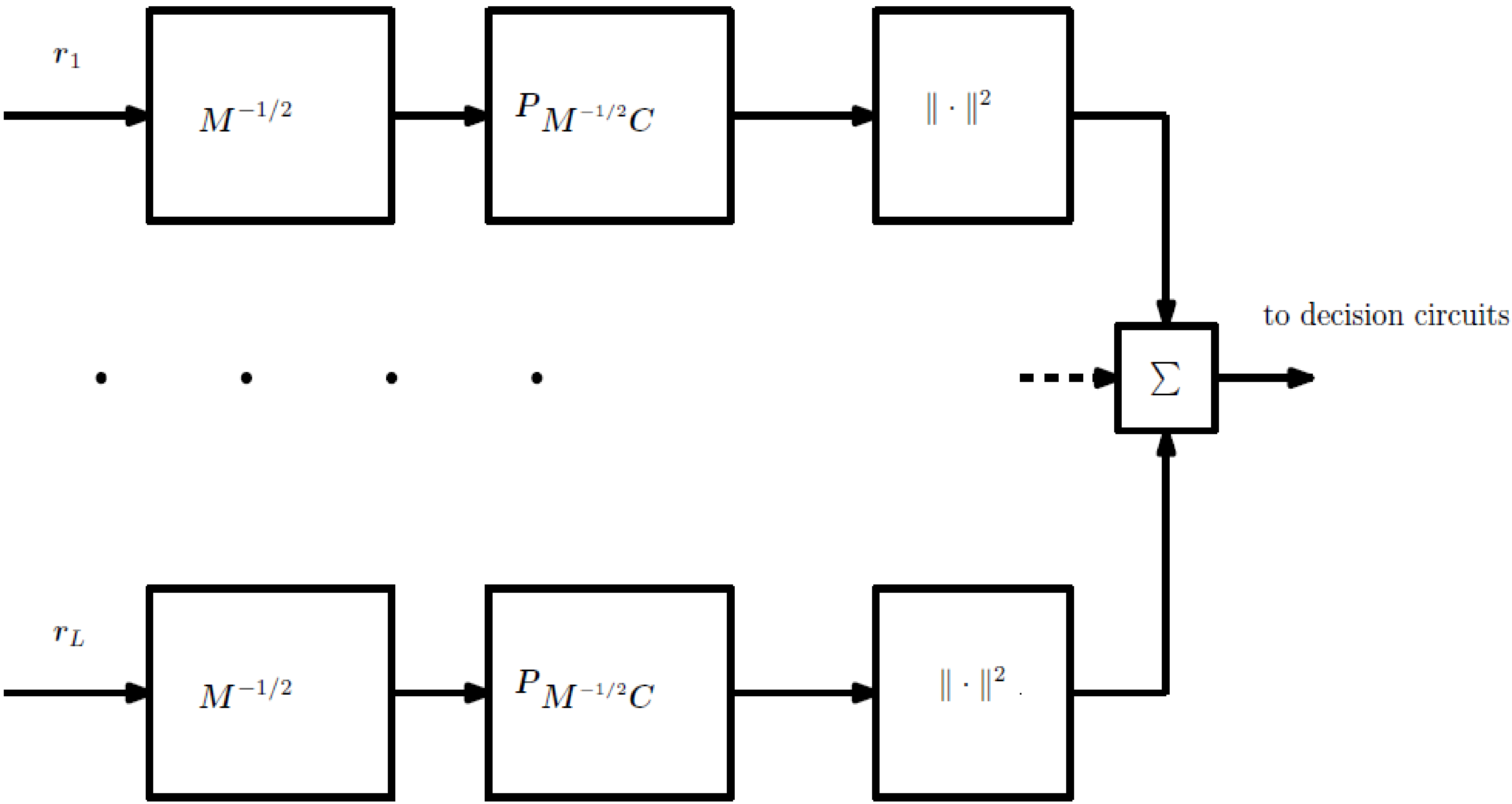
$$\max_{\alpha_1,\ldots,\alpha_M} \frac{f\left(r_1,\ldots,r_L \middle| H_1; M, \alpha_1,\ldots,\alpha_M\right)}{f\left(r_1,\ldots,r_L \middle| H_0\right)} = \sum_{i=1}^{L} \| P_{M^{-1/2}C} r_i \|^2$$

P_V denotes the projector onto the column span of V

If C is full rank, the number of degrees of freedom is $L\min(N,M)$ since:

$$P_{M^{-1/2}C} = \begin{cases} M^{-1/2}C\left(C^H M^{-1} C\right)^{-1} C^H M^{-1/2} & M \le N \\ I_N & M > N \end{cases}$$

Detector structure (un-resolvable paths)



Interesting Special cases

White noise, N=M and orthogonal coding:

GLRT: $\sum_{i=1}^{L} \| \boldsymbol{r}_i \|^2$ (Incoherent integrator on *ML* paths)

Rank-one coding (e.g., $\boldsymbol{C}=\mathbf{1}\mathbf{1}^{\mathrm{T}}$, corresponds to an incoherent power multiplexer)

GLRT: $\sum_{i=1}^{L} |\mathbf{1}^T \boldsymbol{r}_i|^2$ (Incoherent integrator on *L* coherent integrators)

No transmit diversity, only receive diversity, since whatever is transmitted collapses onto the unique direction of the signal space.

Remark: resolvable paths

For resolvable paths, rank-deficient coding would correspond to activating just a subset of the transmit antennas. Once an antenna is activated, it necessarily introduces L diversity paths if the “orthogonality hypothesis under any delay” applies: otherwise stated, if all of the antennas are activated we generate LM diversity paths, but by taking a much larger bandwidth. The optimum detector is obviously an incoherent energy integrator. Thus, un-resolvable paths with full-rank coding achieve the same detection performance as resolvable paths with M active antennas.

Question: can a MIMO radar with widely spaced antennas be considered as a mechanism achieving spatial diversity? Or – rather – it enjoys a form a frequency diversity, possibly combined with angle diversity if the transmitters operate on non-disjoint bandwidths, arising from the wide-band transmission format? This is debatable.

Effect of diversity on detection performance

Assume rank-δ coding under un-resolvable paths (the case of resolvable paths with δ transmitters is similar). The detection perfomance admits the general expression

$$P_{FA} = e^{-\eta} \sum_{k=0}^{\delta L-1} \frac{\eta^k}{k!}$$

$$P_D = \mathrm{E}\left[Q_{\delta L}\left(\sqrt{2\sum_{i=1}^{L} \boldsymbol{\alpha}_i^H \boldsymbol{C}^H \boldsymbol{M}^{-1/2} \boldsymbol{C} \boldsymbol{\alpha}_i}; \sqrt{2\eta}\right)\right]$$

where η is the detection threshold and the generalized Marcum function is defined as

$$Q_K(a;b) = \int_b^{\infty} x \left(\frac{x}{a}\right)^{K-1} \exp\left(-\frac{x^2+a^2}{2}\right) I_{K-1}(ax)\, dx$$

Some comments

Given the detection structure, the signal at the input of the energy detector for RX *i* is

$$z_i = P_{M^{-1/2}C} M^{-1/2} C \alpha_i + P_{M^{-1/2}C} M^{-1/2} w_i$$

As a consequence the conditional (given the scattering) signal-to-interference ratio reads

$$\rho_i = \frac{\alpha_i^H C^H M^{-1/2} P_{M^{-1/2}C} M^{-1/2} C \alpha_i}{\mathrm{Trace}\left(P_{M^{-1/2}C}\right)} = \frac{\alpha_i^H C^H M^{-1} C \alpha_i}{\delta}$$

Remark: the above represents the overall received SNR. There is an inherent trade-off between number of (transmitted) diversity dimensions and overall received signal-to-noise ratio, and rank-1 coding corresponds to concentrating all of the energy along one and the same direction.

Waveform design

- This amounts to choosing the rank δ of the STC matrix (or the number of active transmitters) and to deciding on how to distribute the energy budget among the transmit degrees of freedom.
- Question: does an optimal transmit policy exist? Answer: it depends on the figure of merit. For example, if SNR is the figure of merit, the optimum choice is obviously rank-1 coding along the least interfered direction, which means giving up angular diversity.

Figures of merit

- Figures involving the pair (P_{FA}, P_D);
- The mutual information between the observations available at the receive antennas and the scattering matrix of the target;
- Figures of merit involving Kullback-Leibler divergence;
- Figures of merit involving both the detection performance and the estimation accuracy, if some parameter must be estimated.

Detection and False-Alarm Probabilities

- Here it is necessary to resort to some hypotheses, in order to come up with closed-form results.
- Assumptions: Gaussian scattering;
- Possible tools: maximize the Lower Chernoff Bound (LCB) to the detection probability. Remark, however, that this bound is tight only in the large signal-to-noise ratio region, as detailed later.

LCB

The LCB to the detection probability under Gaussian scattering reads

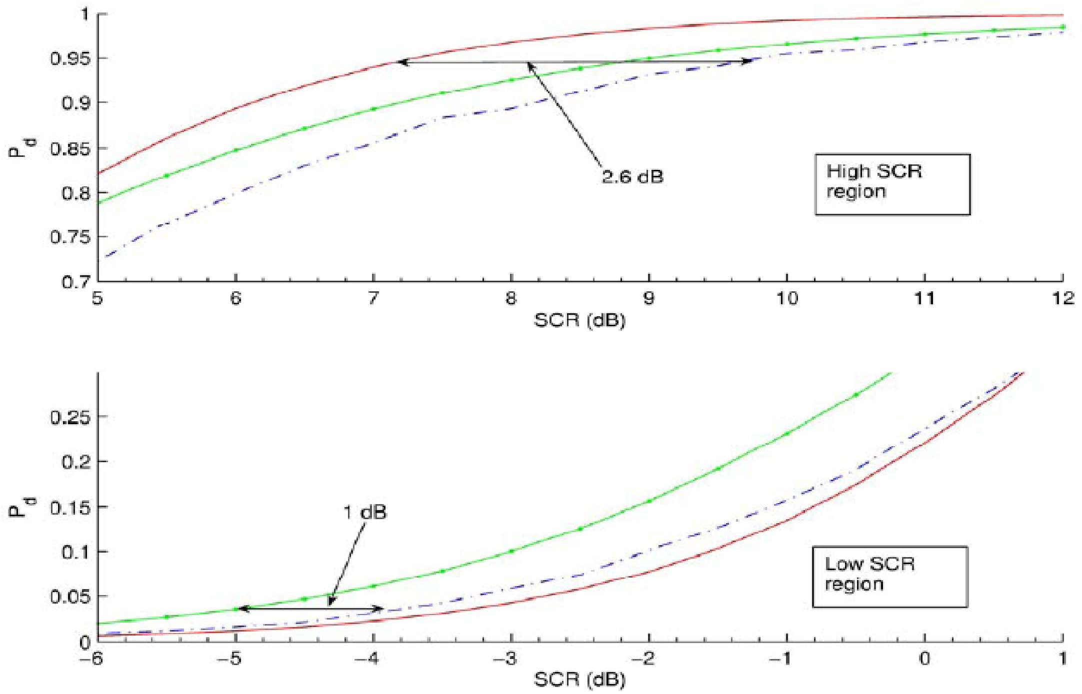
$$P_d \geq 1 - \min_{\gamma \geq 0} e^{\gamma \eta} \prod_{k=1}^{\delta} \left[\frac{1}{1+\gamma(\sigma_a^2 \lambda_k + 1)} \right]^{\delta}$$

where λ_k are the eigenvalues of $\boldsymbol{M}^{-1/2} \boldsymbol{C} \boldsymbol{C}^H \boldsymbol{M}^{-1/2}$

LCB-optimal codes

- Maximize the LCB to the detection probability subject to constraints on the rank of the STC matrix and the average received SNR (much questioned).
- Solution: generate as many independent and identically distributed diversity paths as possible.

Example



N=M=L=2 and white noise

Rank-1 coding and GLRT

LCB-optimal and GLRT

Rank-1 coding and energy detector

Comments

- Apparently, LCB-optimal codes are optimal only in the large SNR region, while rank-deficient coding is largely preferable in the low SNR region.
- This suggests (and theory confirms) that no uniformly optimum transmit policy exists if the figure of merit is detection performance.

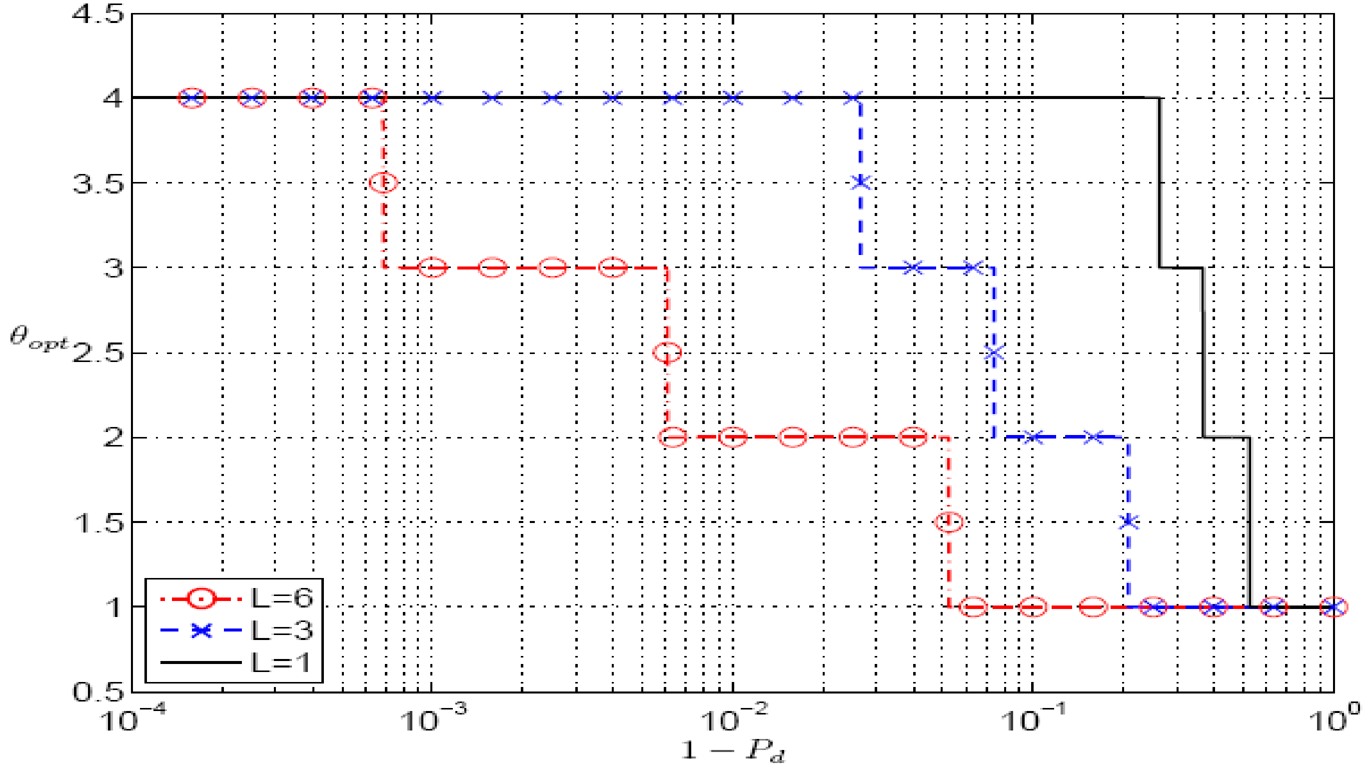
Determining the transmit policy

Find the STC matrix maximizing the LCB subject to:

a) A Semi-definite constraint on the matrix rank, i.e. Rank(C)=$\theta \leq \Delta$
b) Average received energy

The above problem does not admit any solution which is optimal for any SNR, and evidence that an uniformly optimal policy does not exist if LCB is our major figure of merit.

Performance



Optimal rank of the code matrix (i.e., requiring minimum SNR to achieve the miss probability on the abscissas).

$P_{fa}=10^{-4}$; N=M=4;

Different conclusions

Mutual-Information (MI)-optimal codes (Gauss-Gauss and uncorrelated scattering)

$$I\left(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_L; \boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_L\right) = L \sum_{i=1}^{\theta} \log\left[1 + \lambda_i\left(\boldsymbol{M}^{-1/2}\boldsymbol{C}\boldsymbol{C}^H\boldsymbol{M}^{-1/2}\right)\right]$$

Average received SNR:

$$\mathrm{SNR} \propto \mathrm{Trace}\left(\boldsymbol{C}^H\boldsymbol{M}^{-1}\boldsymbol{C}\right)$$

Once again, θ denotes the rank of the STC matrix, and is a design parameter subject to a semi-definite constraint.

Different conclusions

- Subject to semi-definite constraints on the STC rank and the transmit (or receive, when applicable) energy

a) Maximize MI: Solution is diversity maximization, with a power allocation which attempts to create as many independent and identically distributed paths as possible

b) Maximize SNR: Here the solution is rank-1 coding along the least interfered direction.

Comments

- MI is a Schur-concave function of the variables it depends upon;
- Matrix trace is a Schur-convex function of the matrix eigenvalues;
- LCB is neither concave nor convex, but is not tight for low SNR, so it does not appear perfectly suited.

Memo: Schur-convexity/concavity

Let $\boldsymbol{x} \in R^n$ and $\boldsymbol{y} \in R^n$ with $\sum x_i = \sum y_i$

$\boldsymbol{x} \prec \boldsymbol{y}$ if $\boldsymbol{y}$ is "more concentrated", i.e.: $\sum_{i=1}^{m} x_i \leq \sum_{i=1}^{m} y_i, \forall m \leq n-1$

$$\phi(\boldsymbol{x}) \begin{cases} \leq \phi(\boldsymbol{y}) & \phi(\cdot) \text{ is Schur-convex} \\ \geq \phi(\boldsymbol{y}) & \phi(\cdot) \text{ is Schur-concave} \end{cases}$$

Any symmetric (i.e., invariant to argument permutation) function which is concave (convex) is also Schur-concave (Schur-convex). MI is Schur-concave, and attains its maximum when the arguments are maximally "dispersed", i.e. when we generate as many independent and equivalent paths as possible through full-rank coding and water-filling. Trace is Schur-convex and is optimum for maximum "concentration" of its arguments, i.e. Rank-1 coding.

Divergence-based figures of merit

Assume that we have to decide between two hypotheses, corresponding to two densities of the observations, say $f_1(\boldsymbol{x})$ and $f_0(\mathbf{x})$. We know that, the more "different" the densities, the easier to make correct decisions. A measure of such a difference is the pair

$$D\left(f_1 \parallel f_0\right) = D_{10} = \int \log \frac{f_1(\boldsymbol{x})}{f_0(\boldsymbol{x})} f_1(\boldsymbol{x}) d\boldsymbol{x}$$

$$D\left(f_0 \parallel f_1\right) = D_{01} = \int \log \frac{f_0(\boldsymbol{x})}{f_1(\boldsymbol{x})} f_0(\boldsymbol{x}) d\boldsymbol{x}$$

These two measures are relevant in testing f_1 against f_0, whether we resort to Fixed-Sample-Size Tests (FSST) or Sequential Probability Ratio Tests (SPRT).

FSST: Chernoff-Stein Lemma

Let $\mathbf{X}^n$ be iid ~ $f_0(\boldsymbol{x})$ or $f_1(\boldsymbol{x})$ and define P_{FA} as the probability of declaring that $f_1(\boldsymbol{x})$ is in force as the alternative is true, while $P_M=1-P_D$ is the probability of a miss. Then, among all the test achieving $P_{FA}\leq\alpha$, the best that we can do has the following asymptotic property:

$$\lim_n \frac{1}{n} \log P_M = -D_{01}$$

While D_{10} is the error exponent of the false alarm probability. Passing over the details, these two measures also have relevance in SPRT’s, regulating the Average Sample Number (ASN) needed to make decisions under the two hypotheses.

Sequential tests

Here the number of observations is random and any performance can be obtained by collecting a large enough number of samples. The primary parameters are thus the average sample numbers (ASN's) under the two alternatives. Sequential Probability Ratio Test (SPRT) aimed at discriminating between $f_0(\boldsymbol{x})$ and $f_1(\boldsymbol{x})$ enjoys the following:

$$ASN_{H_0} \propto \frac{1}{D_{01}}$$

$$ASN_{H_1} \propto \frac{1}{D_{10}}$$

Figures of merit

Divergence-based figures of merit have been proposed in

Kay, *IEEE Trans, AES*, 2009, D_{01} adopted as foM
Tang, Li, Wu, *IEEE Signal Processing Letters*, 2009,
Grossi, Lops, *IEEE Trans. Info. Theory*, 2012, proposing the convex combination $\kappa D_{10}+(1-\kappa)D_{01}$

In general, it is interesting to define, as a function of the waveforms that can be transmitted (i.e., in our setup, of the STC matrix) the achievability region, by constraining either the transmit energy or the received SNR. This gives us all of the information on what can be achieved, as far as detection is concerned, through STC in MIMO radar.

Achievability region, constrained received SNR

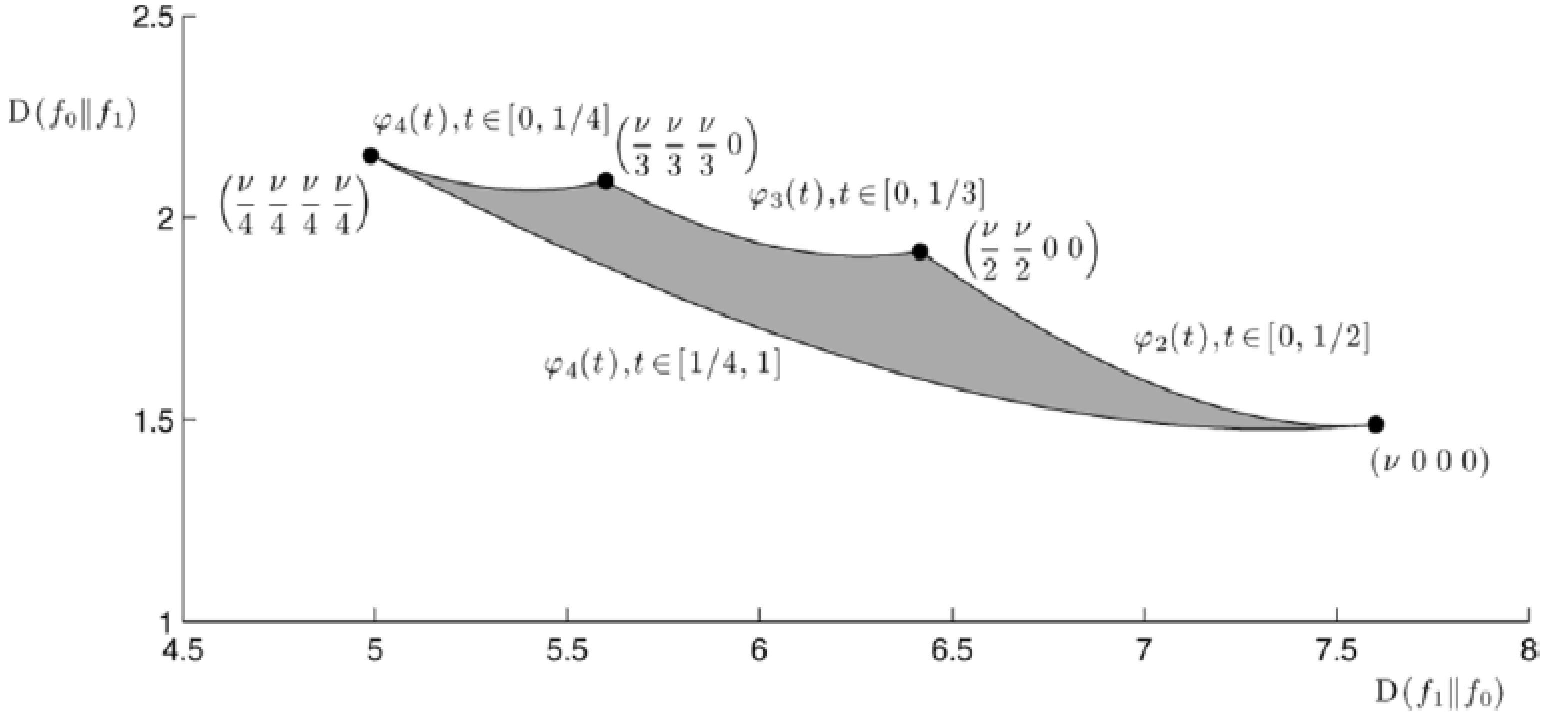


Fig. 1. Achievable region of divergence pairs when $\Delta = 4$ and the SNR is constrained to $\nu = 10$ dB.

Achievability region, constrained received SNR

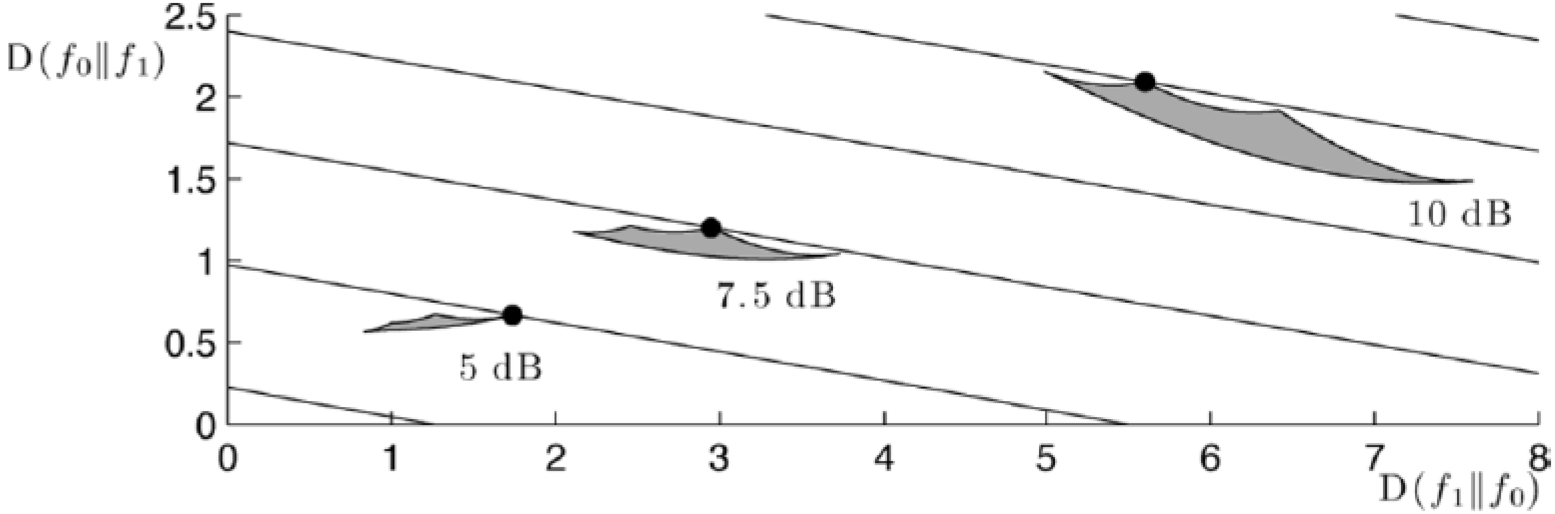
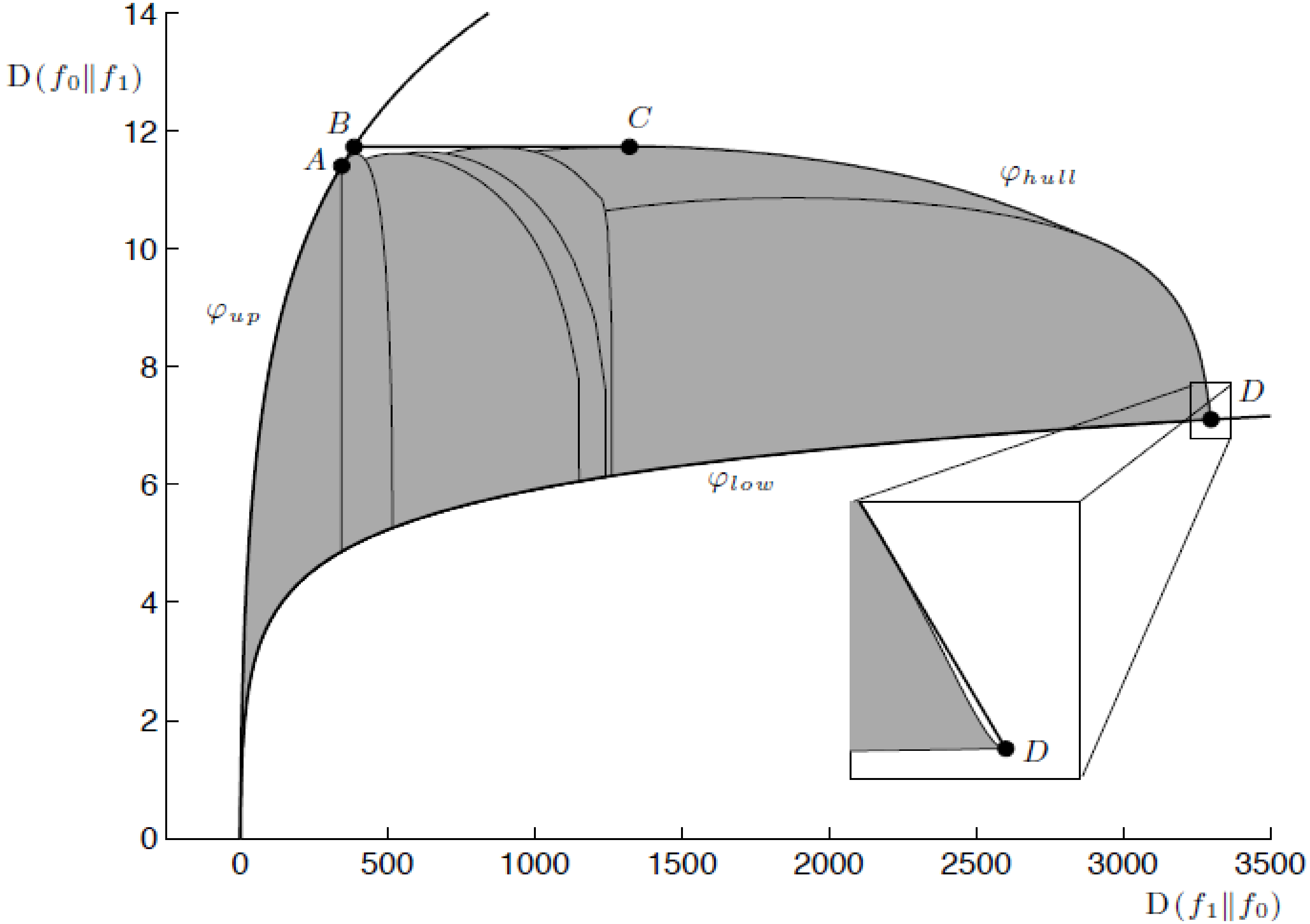
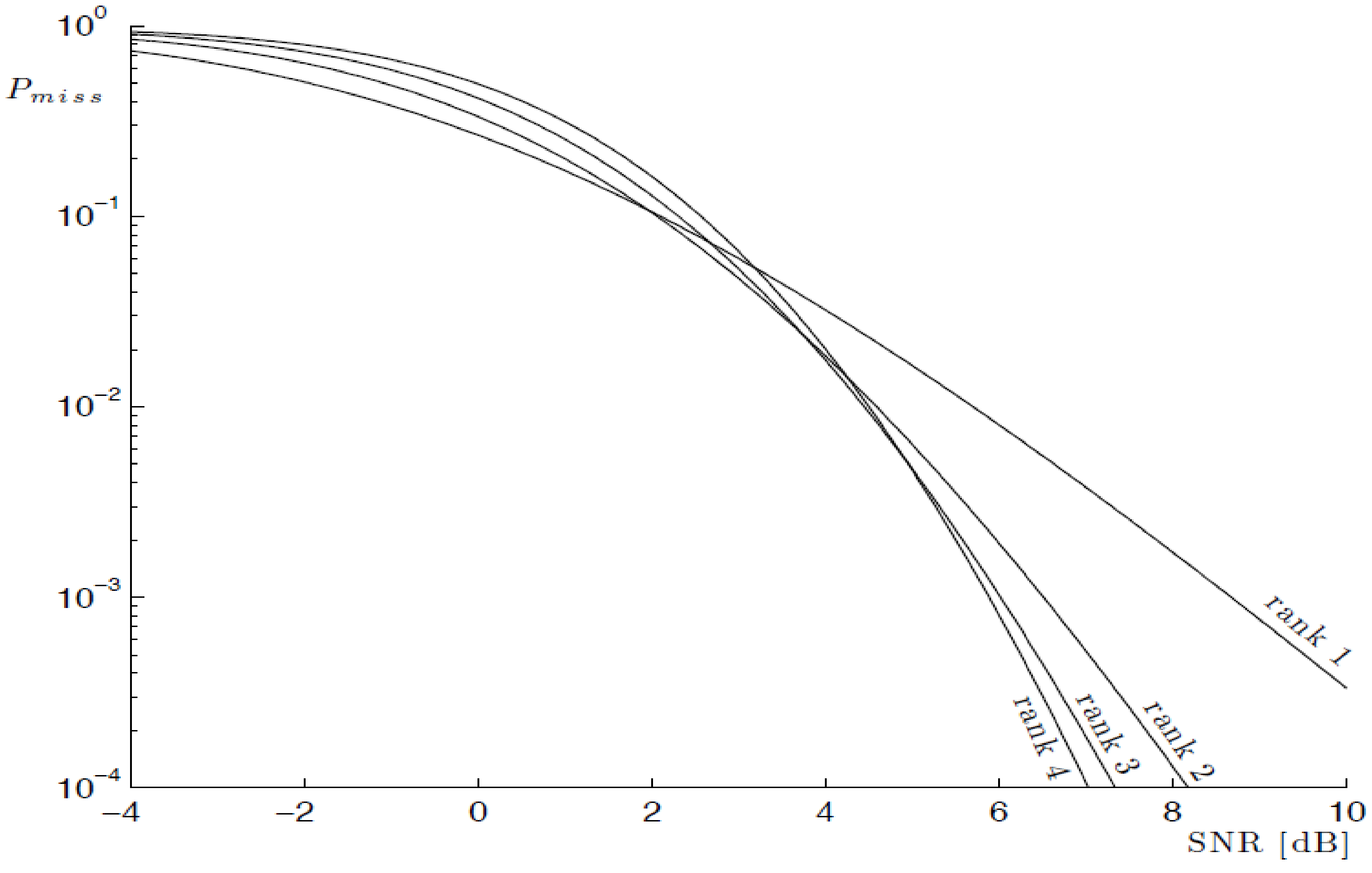


Fig. 2. Achievable regions of divergence pairs when $\Delta = 4$ and the SNR is constrained to $\nu = 5, 7.5, 10$ dB. The parallel lines are the level sets of the merit function in (6) with $\kappa = 0.15$, and the markers denotes the corresponding optimal points (i.e., the solution to Problem 1): the optimal coding rank is 1 for 5 dB, 2 for 7.5 dB, and 3 for 10 dB.

Achievability region, constrained transmit energy

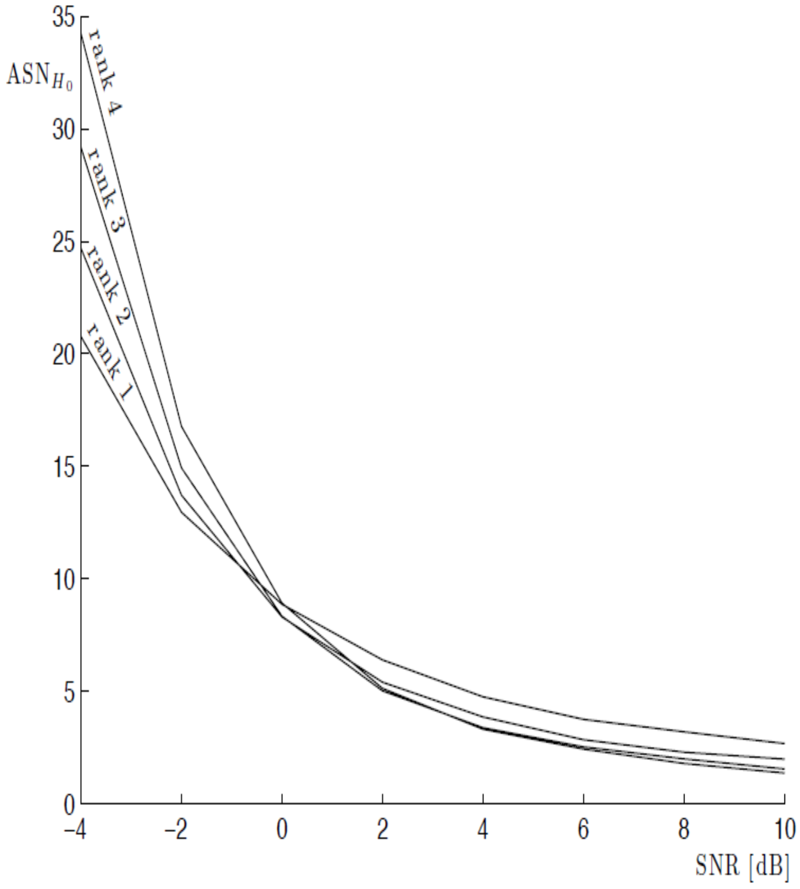


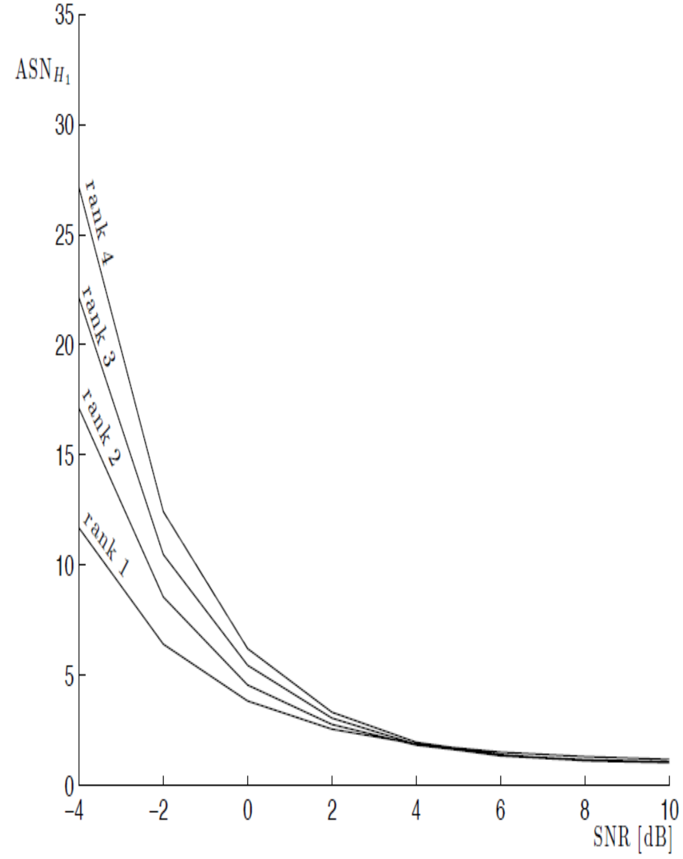
An example of application: FSST



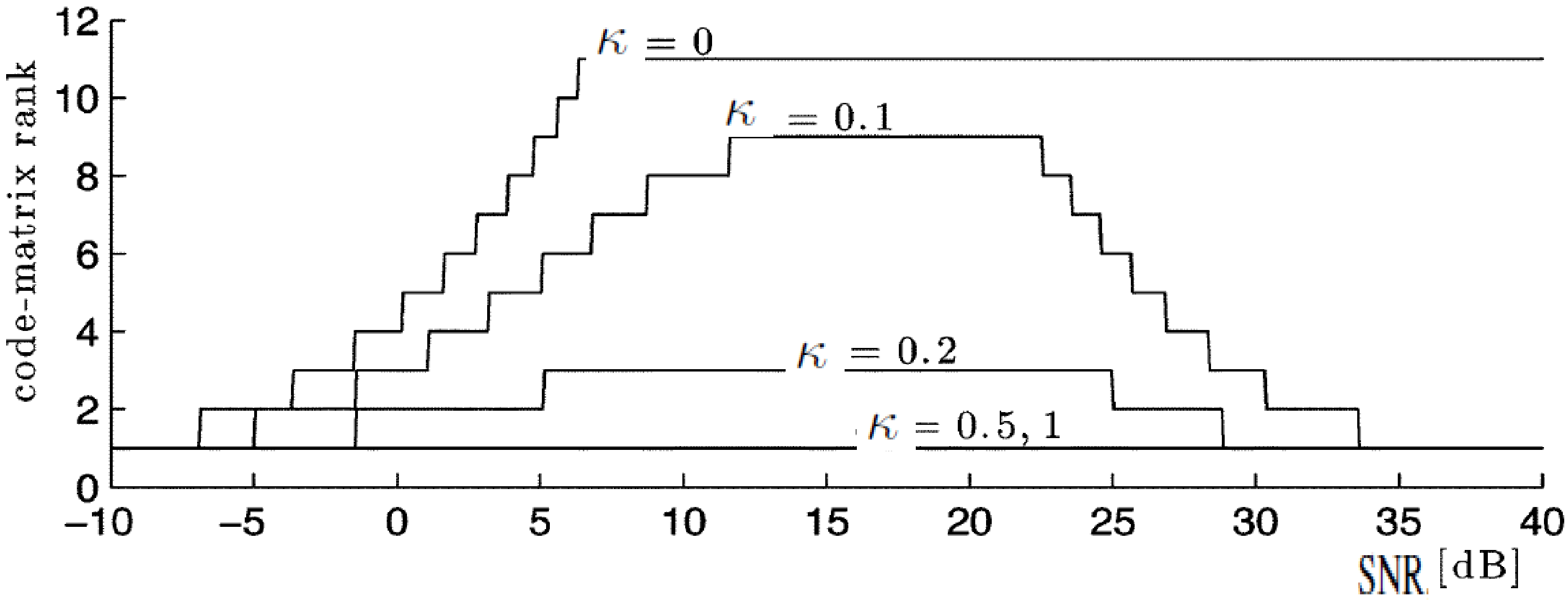
FSST, divergences chosen based on the achievability region.

An example of application: SPRT





Synopsis (Δ=11)



The evidence speaks in favor of the fact that, in the low SNR region, we'd better use MIMO as an incoherent power multiplexer rather than as a source of diversity (or, equivalently, that we concentrate all of the power on one transmitter). $\kappa=0$ corresponds to assuming D_{01} as a measure of performance, and $\kappa=1$ to assuming D_{10}.

Question

Is there any realistic situation – concerning detection – wherein full angular diversity is desirable? Otherwise stated, if full diversity is advantageous only in the large SNR region, why to pay so much attention to it?

A possible answer resides in the fact that, so far, we have operated under fairly ideal conditions, studying the fundamental limits with scarce attention to a such a basic issue as robustness, i.e. resilience of the detection performance with respect to deviations between the nominal and the actual operating conditions. These deviations may concern:

- The disturbance covariance;
- The amount of transmit diversity (we underline here that, for given wavelength, path independence is a function of the distance and of the target extension);
- The amount of receive diversity (same situation as above).

It is thus interesting to focus attention on the issue of robust waveform design, which leads us to fairly different conclusions!!

Theoretical framework

Let $\mathbf{M}_w = \boldsymbol{Q}_w \otimes \boldsymbol{R}_w$ be, as usual, the disturbance covariance and $E[\boldsymbol{\alpha}_i \boldsymbol{\alpha}_i^H] = \boldsymbol{Q}_\alpha \otimes \mathbf{R}_\alpha$ the clutter and scattering covariance, encapsulating the transmit diversity (matrices $\boldsymbol{R}$) and the receive diversity (matrices $\boldsymbol{Q}$, assumed $\boldsymbol{I}_L$ for simplicity). The STC matrix **C** (or transmitted waveform) can be "robustified" under two different situations through the mini-max principle

Let $\lambda\left(M^{-1/2} C R_\alpha C^H M^{-1/2}\right)$ the set of (at most Δ) eigenvalues of the matrix in brackets

Let $f\left[\lambda\left(M^{-1/2} C R_\alpha C^H M^{-1/2}\right)\right]$ a cost function depending on these eigenvalues

Known noise covariance: $$C = \arg\min_C \max_{R_\alpha} f\left[\lambda\left(M^{-1/2} C R_\alpha C^H M^{-1/2}\right)\right]$$

Unknown noise covariance: $$C = \arg\min_C \max_{R_\alpha, M} f\left[\lambda\left(M^{-1/2} C R_\alpha C^H M^{-1/2}\right)\right]$$

Mini-max Waveforms

All cost functions which are decreasing and Schur-convex admit one and the same robust solution, i.e. full transmit diversity. The power allocation depends on the operating conditions, i.e.:

- Water-filling, if **M** is known, assigning more power to the least interfered directions;
- Isotropic transmission, if **M** is unknown.

Interpretation: No empty sub-space exists where the target may hide. What matters here is the transmit policy (i.e., the amount of transmit diversity), while the power allocation strategy is definitely less critical.

Problem: Do cost functions fulfilling the above conditions have any relevance in radar detection/estimation?

Admissible cost functions

Are in the required form the following families of cost functions

- ❑Any decreasing function of the received SCR;
- ❑The LMMSE estimator of the target scattering matrix;
- ❑The mutual Information between the observations and the target scattering.

Moreover, since:

$$P_M \simeq \begin{cases} 1 - P_{FA} - LN\left(P_{FA} + \frac{e^{-\eta}\eta^{ML}}{(ML)!}\right) SCR & \text{(Low SCR region)} \\ \exp\left[\gamma\eta - L\sum_{i=1}^{\Delta} \ln\left(1 + \gamma(1+\lambda_i)\right)\right] & \text{(Chernoff bound for the large SCR region)} \end{cases}$$

also the probability of miss will do, at least in the two relevant regions of large and small SCR.

Remark

The previous derivations assumed full receive diversity and partially or totally unknown transmit diversity. Everything carries over to the general case where both scattering and clutter have unknown transmit and receive diversity, and the major results re-apply to the cases where the overall covariance matrices take on the form of a Kronecker product between transmit and receive covariance matrices.

Example of application: effect of transmit diversity

M=L=2 (Number of receive and transmit antennas), N=4 (code-word length)

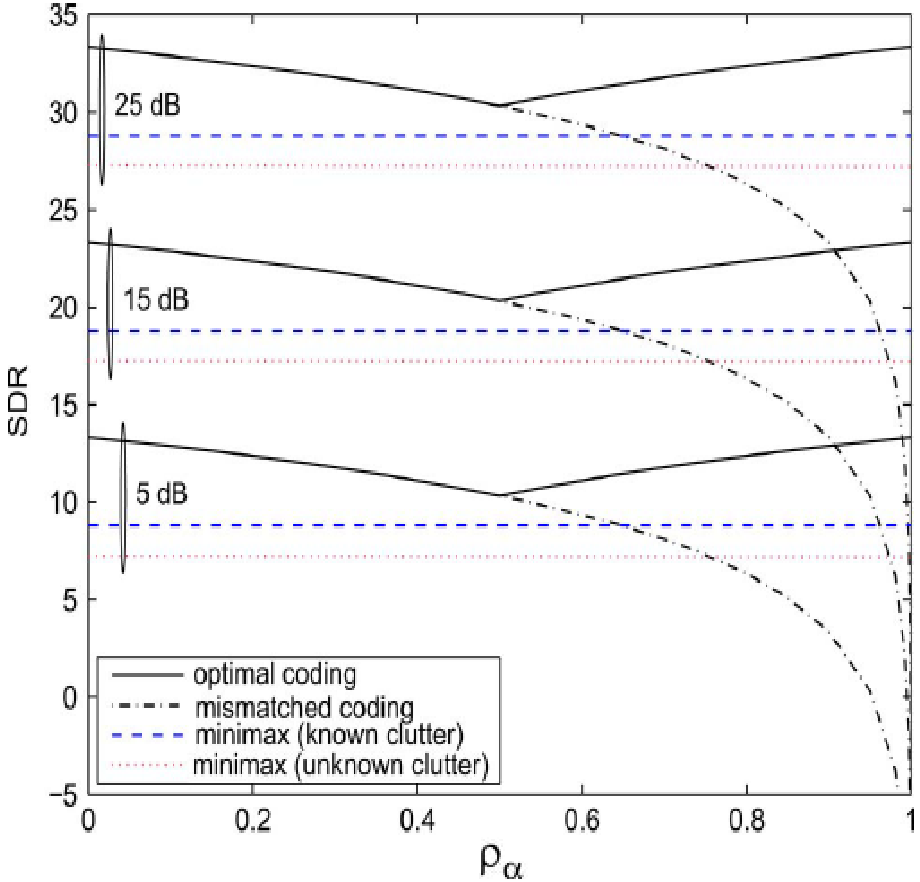
$\boldsymbol{R}_{\alpha}=\boldsymbol{U}_{\alpha}\,\boldsymbol{\Lambda}_{\alpha}\,\boldsymbol{V}_{\alpha}^{H}$, $\boldsymbol{\Lambda}_{\alpha}=\sigma^{2}_{\alpha}\mathrm{diag}(\rho_{\alpha},1-\rho_{\alpha})$

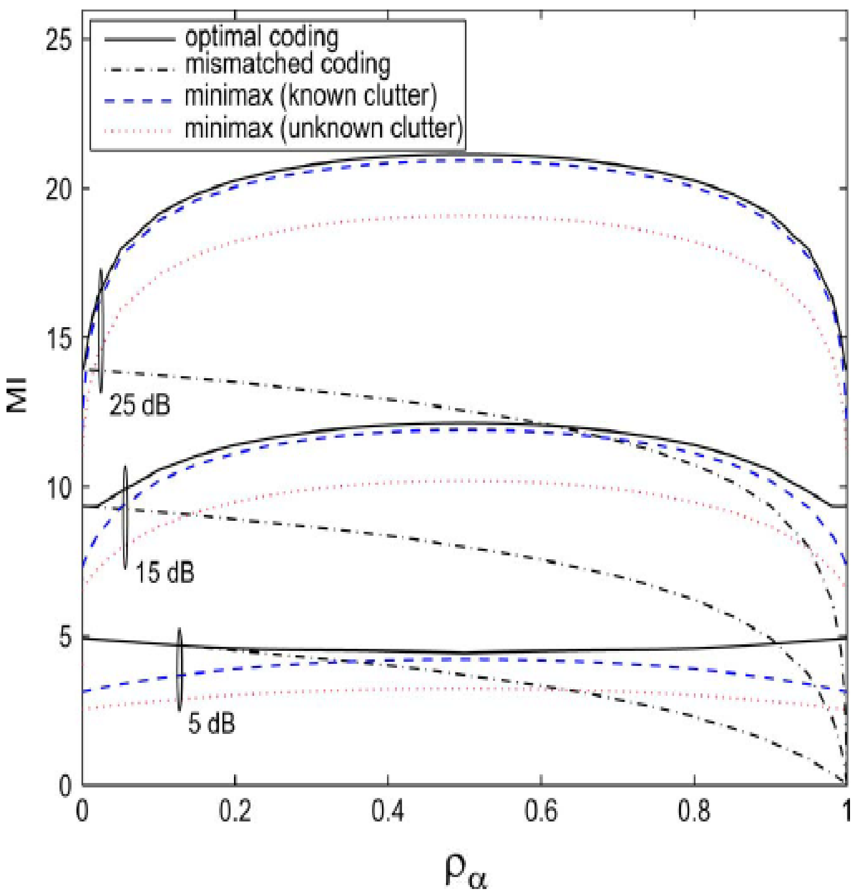
$\boldsymbol{R}_{w}=\boldsymbol{U}_{w}\,\boldsymbol{\Lambda}_{w}\,\boldsymbol{V}_{w}^{H}$

Legenda for next slides:

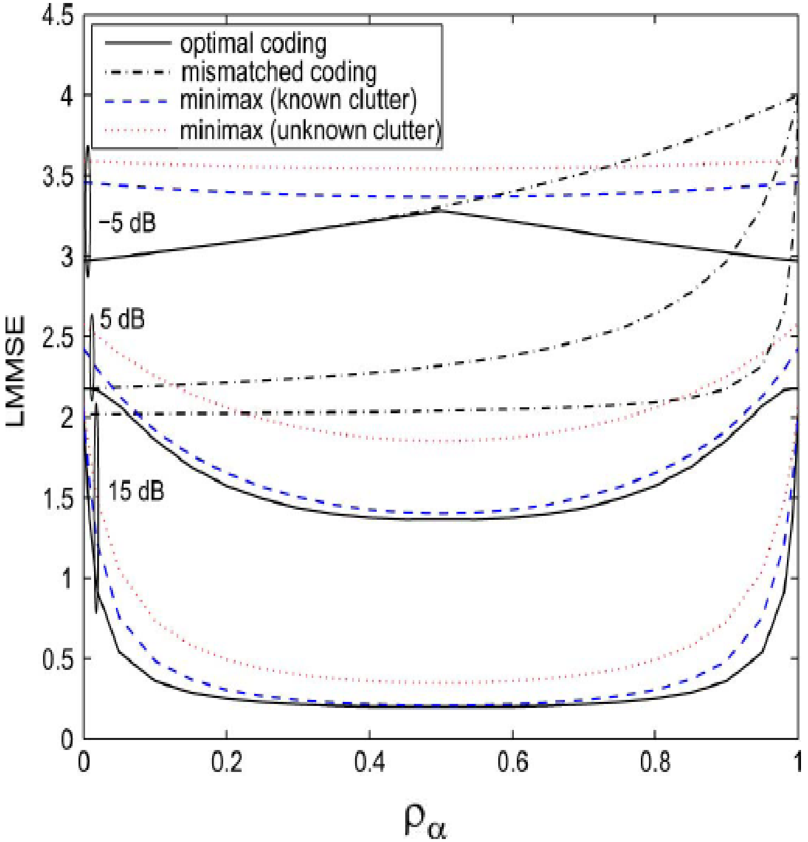
Optimal Coding: everything is known;
Mismatched coding: the receiver assumes $\rho_{\alpha}=0$, and the signal is built around the absence of transmit diversity.

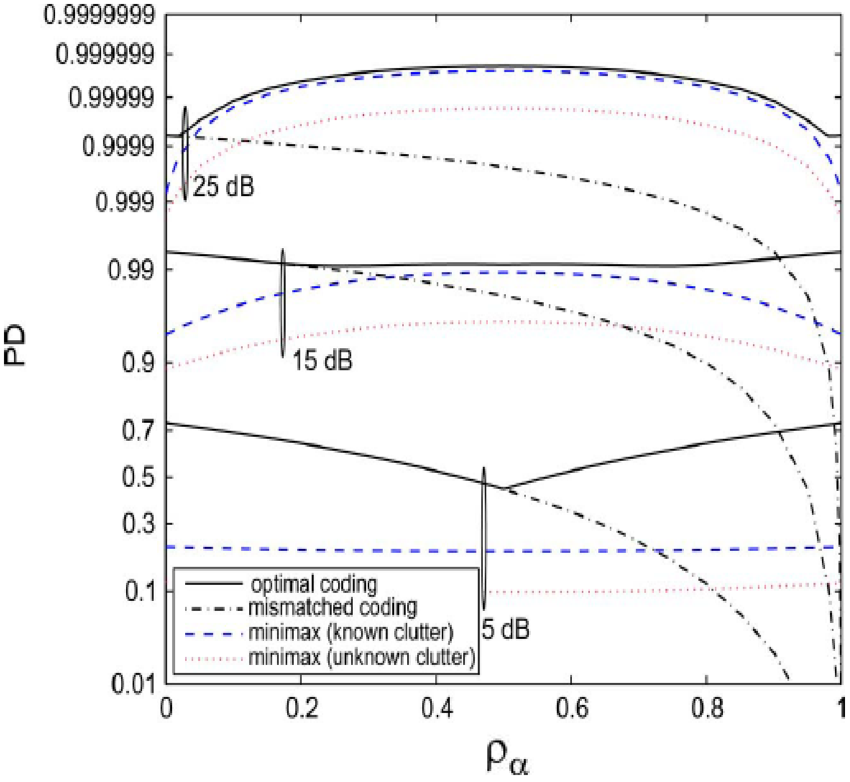
The typical figures of merit





More figures of merit





Major role of (angle) diversity in detection

At the transmitter end: full angle diversity, which may be achieved through full-rank coding for un-resolvable paths or by distributing the available power among a number of wide-band transmitters, for the case of resolvable paths, is fundamental in granting robustness in almost all of the most credited figures of merit.

At the receiver end, of course diversity is always beneficial, which is a well-known results concerning decentralized systems.

The above holds true for detection (and, somehow, parameter estimation if LMMSE is considered). What about other issues like localization? This is a long story, and we do not have much time, but we can give some general trends.

What about localization?

- Two main situations:
  - Un-resolvable paths: target ranging
  - Resolvable paths: target localization through multi-lateration.

The latter has been thoroughly studied in

Godrich, Haimovich and Blum, “Target Localization accuracy gain in MIMO radar-based systems”, *IEEE Trans*. *Info*. *Theory*, 2010.

Godrich, Petropolu, Poor, “Power Allocation Strategies for Target Localization in Distributed Multiple-Radar Architectures”, *IEEE Trans*. *Signal Processing*, 2011.

Target Ranging in un-resolvable paths

Assume a space-time coded MIMO system, transmitting the waveforms

$$s_i(t) = \sum_{n=1}^{N} c_{i,n}\psi_n(t)\,, \quad i = 1, \ldots, M$$

$$\boldsymbol{s}(t) = \begin{pmatrix} s_1(t) \\ s_2(t) \\ \cdots \\ s_M(t) \end{pmatrix} = \boldsymbol{C}^T \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \cdots \\ \psi_N(t) \end{pmatrix}$$

$$C^T = U \Sigma V\,, \qquad \Sigma = \mathrm{diag}\left(\lambda_1, \ldots, \lambda_M\right)$$

Fisher information in delay estimation

$$\boldsymbol{r}(t) = \boldsymbol{H}\boldsymbol{s}(t-\tau) + \boldsymbol{w}(t), \qquad \boldsymbol{w}(t) \text{ white and } \boldsymbol{H} \text{ Complex Normal}$$

$$I_\tau = 2L\Omega^2_{\text{rms}} \sum_{i=1}^{M} \frac{\frac{\lambda_i^2}{N_0}}{1+\frac{\lambda_i^2}{N_0}} \qquad \Omega^2_{\text{rms}} = (2\pi)^2 \int f^2 |\Psi_1(f)|^2 \, df$$

This is a Schur-convex function, which is maximum for rank-1 coding.

Consequence

If $f\,(\boldsymbol{\Sigma})$ is any figure of merit, no matter its Schur-concavity/convexity characteristic, an optimization problem in the form

$$\begin{cases} \max_{\boldsymbol{\Sigma}} f\left(\boldsymbol{\Sigma}\right) \quad \text{subject to} \\ I_\tau \geq \beta \end{cases}$$

Cannot but imply rank-deficient coding. In other words, angle diversity maximization is not optimal except that if the figure of merit is Schur-concave *and* the constraint marginal (i.e., the value of β is small).

Resolvable paths (M×L MIMO)

Localization in a plane of a target located at (*x*,*y*);
Deterministic channels.

Facts:

- the (measurable) delays $\tau_{i,j}$ are known functions of (*x*,*y*), once the TX and RX positions are known.
- Any vector parameter $\boldsymbol{\theta}$ containing (x,y) as its first two entries has FIM $\boldsymbol{J}(\boldsymbol{\theta})$.
- The 2×2 north-western sub-matrix of $\boldsymbol{J}^{-1}(\boldsymbol{\theta})$, say $\boldsymbol{D}(\boldsymbol{\theta})$, has, on its diagonal elements, the CRLB to the estimates of *x* and *y*.
- The key cost function is thus Trace($\boldsymbol{D}$).

For details, see Godrich, Haimovich, Blum, *IEEE Transactions on Information Theory*, 2010.

Design parameters and constraints

$$\boldsymbol{p} = \begin{pmatrix} p_1 \\ p_2 \\ \dots \\ p_M \end{pmatrix}$$ Vector of the powers assigned to the available transmitters

$$\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_M \end{pmatrix}$$ Vector of the bandwidths assigned to the available transmitters

$p^T 1 = \sum_i p_i \leq P$ Total available power

$w^T 1 = \sum_i w_i \leq B$ Total available bandwidth

$\mathrm{Trace}\,(D) = f\,(\boldsymbol{p}, \boldsymbol{w})$ Cost function

Some allocation criteria

Uniform power (bandwidth) allocation : $p_i = \frac{P}{M}$ $\left(w_i = \frac{B}{M}\right)$

Optimum power allocation : $\begin{cases} p_{\text{opt}} = \arg\max_{\mathbf{p}} f\left(p; \frac{B}{M}\mathbf{1}\right) & \text{subject to} \\ p^T\mathbf{1} \leq P\,, p_i \geq 0 \end{cases}$

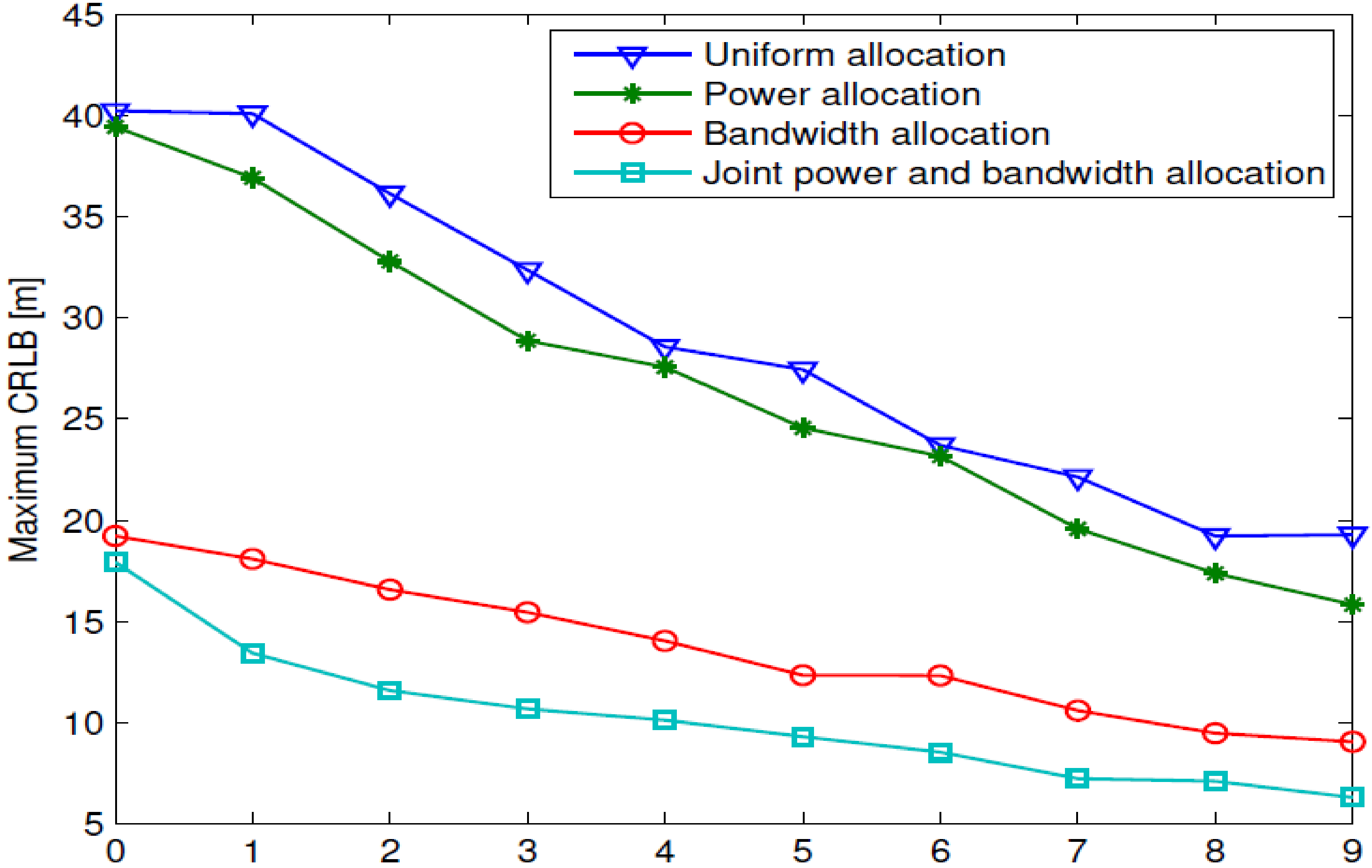
Optimum bandwidth allocation : $\begin{cases} w_{\text{opt}} = \arg\max_{\mathbf{w}} f\left(\frac{P}{M}\mathbf{1}; w\right) & \text{subject to} \\ w^T\mathbf{1} \leq B\,, w_i \geq 0 \end{cases}$

Optimum power and bandwidth allocation : $\begin{cases} \left(p_{\text{opt}}, w_{\text{opt}}\right) = \arg\max_{\mathbf{p},\mathbf{w}} f\left(p; w\right) & \text{subject to} \\ p^T\mathbf{1} \leq P\,, w^T\mathbf{1} \leq B\,, p_i \geq 0\,, w_i \geq 0 \end{cases}$

Example

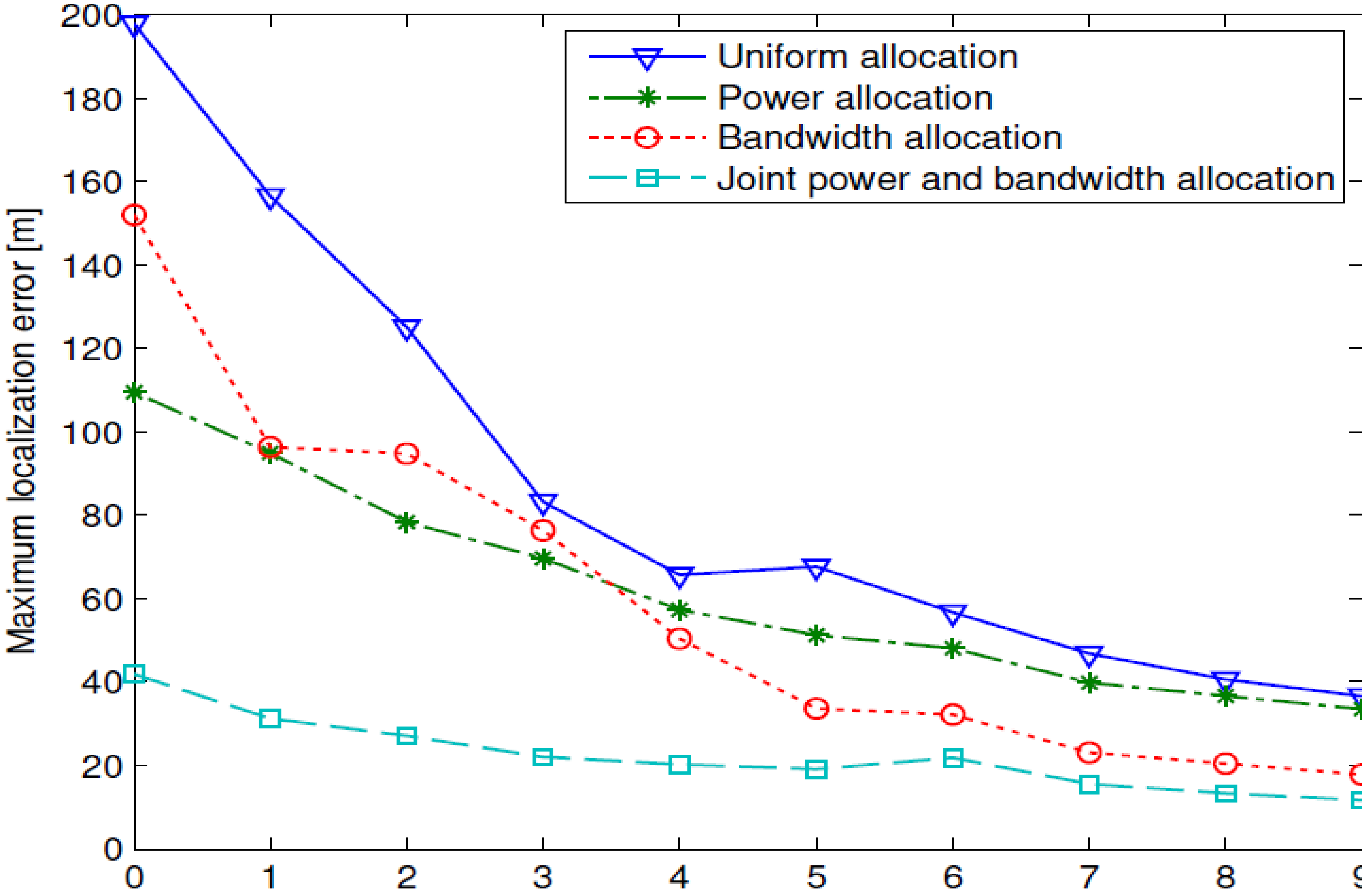
- 5 transmitters and 5 receivers
- Available bandwidth: 3 MHz
- PRT= 5 kHz
- 4 stationary equal-strength targets with RCS=10m^2;
- The targets are randomly located in a 20km×20km area
- The channels are Rayleigh and random.
- 1000 trials

The cost function



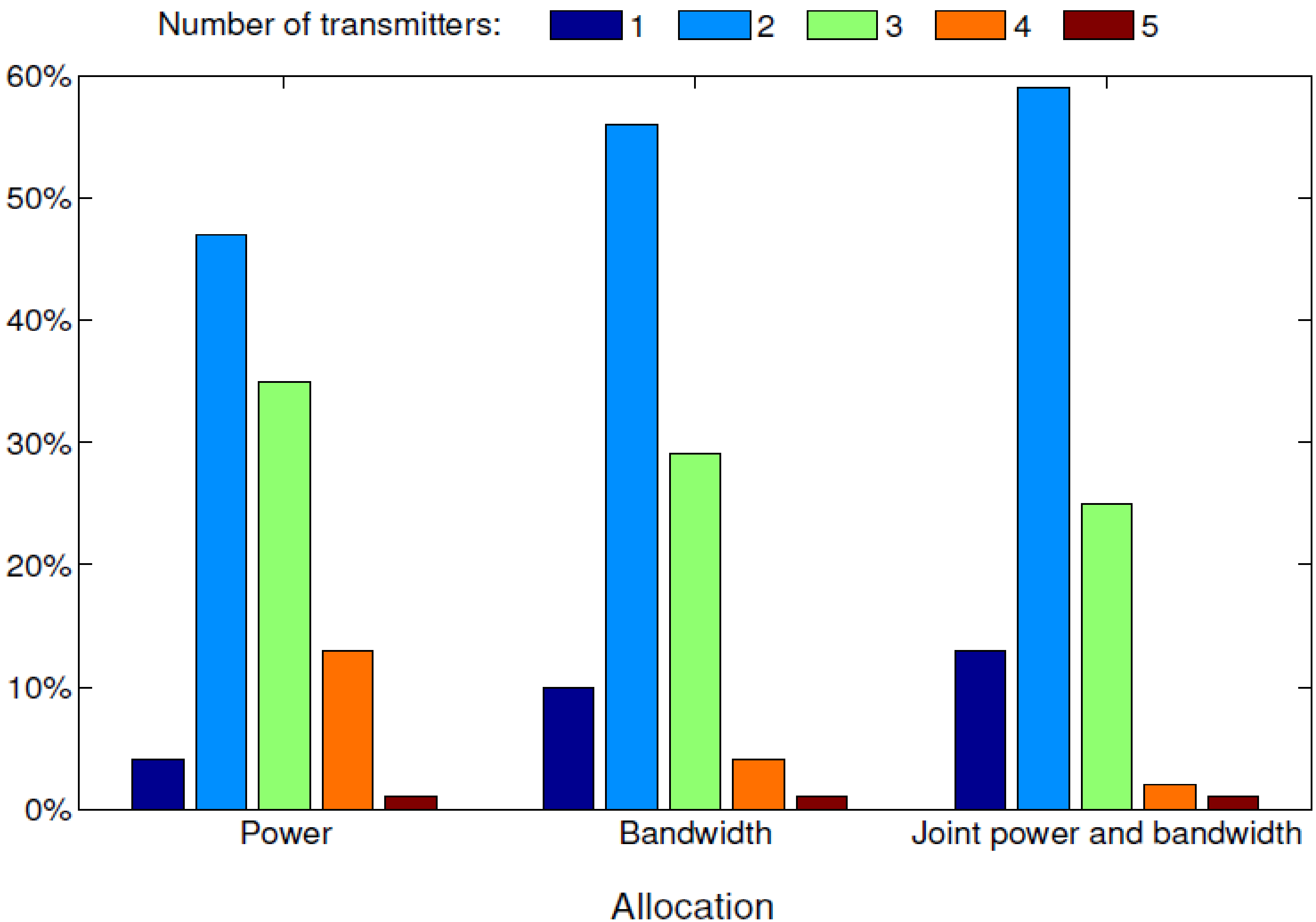
Square root maximum CRLB vs. SNR after resource allocation.

Achieved performance

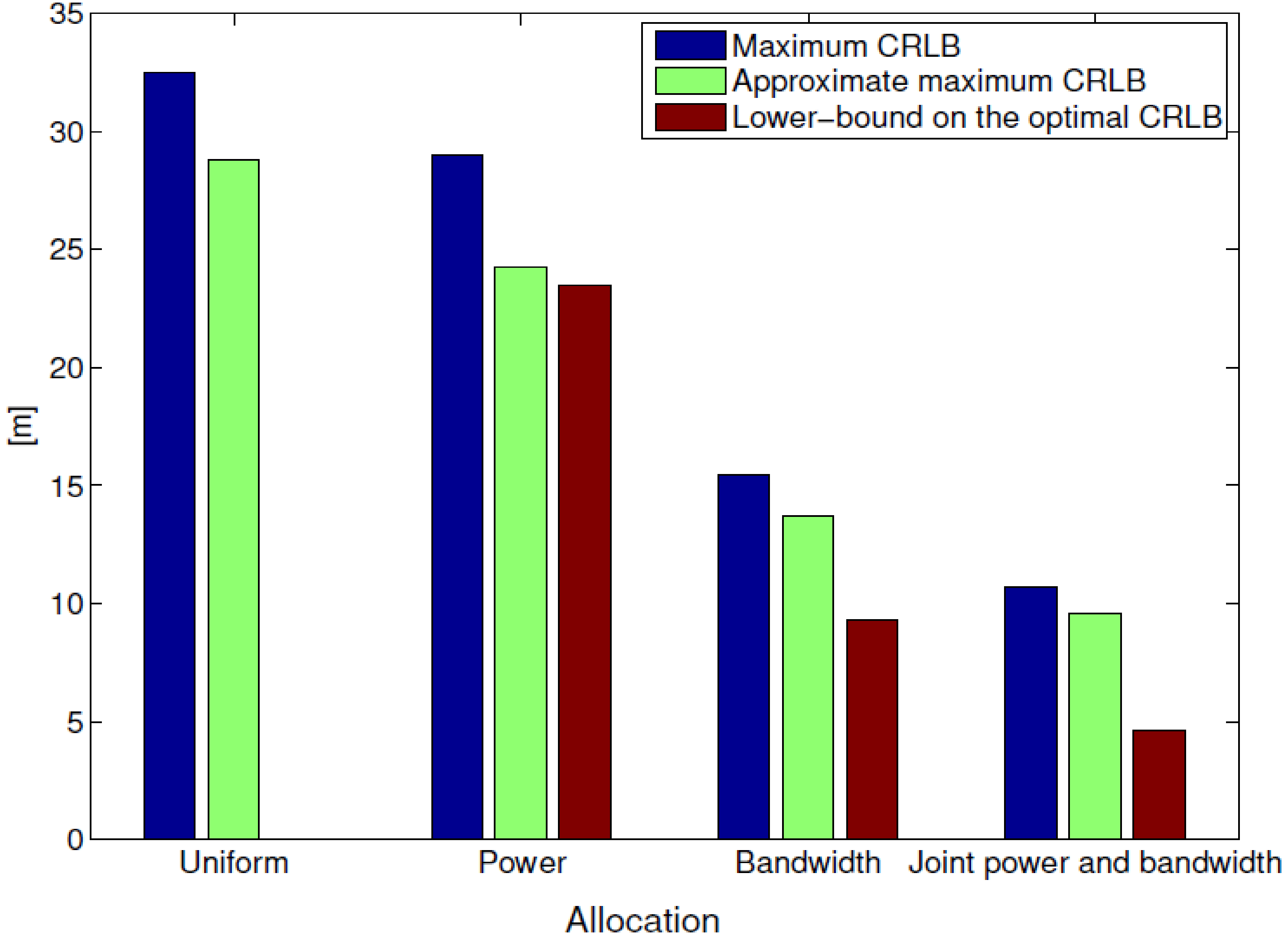


Square root maximum localization error vs. SNR after resource allocation.

Resolvable paths: amount of transmit diversity



Resolvable paths: effect of the optimization



Some comments

- The previous plots refers to un-equal average SNR, each channel being characterized by a Rayleigh attenuation with unit mean square value;

- Once again angular diversity appears more a means for robustifying localization procedure than a tool to improve performance, as clearly demonstrated by the fact that the “best” (i.e., most frequently chosen) transmit policy does not rely on transmit diversity maximization.

- A possible conclusion is that, for both detection and localization, the amount of transmit diversity must be a compromise between optimality and robustness, on the understanding that in many application maximizing the transmit diversity is a mini-max (or maxi-min, if a merit function is involved) choice.

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Remark: the last is kind of critical with respect to angular diversity, but offers an interesting point of view.

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